

Structural analysis and modeling of Georgian and Medieval polyphonies

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The similarity of traditional Georgian singing with late-medieval Western European music has been a trigger of speculations about possible relationships between both for a long time. Today, it is commonly assumed that traditional Georgian polyphonic singing and Western European notated polyphony have developed independently of each other (e. g. Jordania, 2006; 2010). In the present paper, we revisit this issue from an exploratory perspective, using structural analysis and modeling of a small corpus of three-voiced Georgian and Medieval songs. Our analysis proceeded in two ways. The first approach uses a classical musicological perspective to distinguish the „pillars“ and the „ornaments“ in the harmonic structure of the songs and subsequently study their temporal development following Arom (2017). The second approach uses a representation of songs as directed graphs (Scherbaum et al., 2015; 2016), which provides an intuitive framework for the graphical comparison of individual scores and for the analysis of the effect of reducing a score to its “pillars”.

CLASSICAL MUSICOLOGICAL PERSPECTIVE

The trajectory of any Georgian polyphonic song includes a series of *irregularly distributed vertical conjunctions* – traditionally separated by improvised passages –, which define its particular nature: these are the pillars which form its *matrix*; they constitute the mental reference, the *cognitive scheme* which all of the singers carry in their memories.

Many aspects of Georgian traditional polyphony – particularly historical and ethnological aspects – have been studied, mainly by Georgian and Russian researchers. However, the underlying principles of its grammar have never been addressed in a systematic way. This is what led Simha Arom and Polo Vallejo, in 2007, to undertake the study of the harmonic syntax of this music.

Modeling and Models

The contrapuntal complexity of the songs and the multiplicity of the chords that appear in them made it necessary to present them in a simplified form. This implied *modeling*. Modeling allows us to grasp the relations prevailing between the spontaneous production of a musical event and the idea it springs from. The modeling activity is not necessarily limited to the reconstituting of concrete objects, such as a given piece or repertoire. The same can aim to explore, or even to reconstruct, certain properties of the components revealed by the analysis work.

By *model*, we mean “*a representation, both overall and simplified, of a musical entity. The model condenses, in outline form, all of this entity's distinctive features and no others, thus revealing its uniqueness*” (Arom, 1991). The model is thus equivalent to the barest realization of a piece that can be identified as such by the bearers of the tradition to which it belongs. It is precisely what preserves the identity of a piece of music and allows for its oral transmission.

Georgian polyphony uses all seven diatonic modes. While many songs use one single mode, some contain transitions to one or several other modes. The latter necessarily occur within a segment of a piece¹, and we had therefore to determine their respective modes.

To do this, we chose one single criterion: the *finalis* of any segment or song will always be considered the first degree of its mode. This criterion – even though it may seem somewhat arbitrary – is very useful, since it allows for the establishing of a point of reference which is coherent not only for a set of pieces, but for the whole Georgian polyphonic corpus. It is important to emphasize that the attribution of a number to the various degrees of any mode is the *indispensable condition* for being able to label the chords based on these degrees, *independent of the mode in which they appear*.

During the analysis, it appeared that all of the chords that occur in the songs fall into two distinct categories: first, those which are seemingly random and result solely from the movements of the voices and secondly those which have a structural function. At this stage, our goal was to determine the *harmonic framework* of each musical entity, in other words, to detect, among the numerous chords of a piece or a segment, those which, beyond the different realizations of any song, *remain stable* and ensure its identity.

What is the harmonic framework? It is the series of *fixed chords* distributed over the course of each piece and which are mostly separated by *brief improvised sequences*. As a reminder, these “pillars” constitute its *matrix*, or *cognitive scheme*, which is present in the background of any of its realizations.

In order to materialize this matrix, it was essential to determine for each chord whether or not it falls within the harmonic framework. This implies the following operations: *interactive experimentation*, *modeling* and *validation*, all of which must be carried out with the collaboration of local singers with recognized experience.

To arrive at this “reduction” of a musical entity, we eliminated the passing tones, neighboring and escape tones, appoggiaturas, anticipations and suspensions, and also – when the harmony remains the same –, the changes of position of sounds within the same chords.

As outsiders to the culture, one could not be certain that the result of such an approach would be more than purely speculative. In order to validate it culturally, we had to call on local experts. We thus benefited from the invaluable help of the members of the Georgian State Vocal Ensemble *Basiani*: after explaining our objective to them, we asked them to perform the pieces that had been reduced by us, but under the following conditions:

- removing the words,
- keeping only the chords that seemed to them to constitute the pillars of the song,
- scrupulously respecting the time interval that separates each of these chords from those which precede and/or follow them, so that the proportions of the durations in the “real” song are not affected.

Modeling is the key step in the processes of analysis and validation. It represents the end point of any analytic approach, and it is also the starting point of the procedure by which the analysis can be validated.

Relationships between Written and Oral Polyphonies

Many musicological studies have suggested relationships between written polyphony and oral polyphony, which have persisted since the High Middle Ages. It was known as of the beginning of musical notation, and still attested in the middle of the eighteenth century.

Many studies published over the past fifty years stress the place and the role of improvisation in the music of the Middle Ages and the Renaissance. It indeed seems, in light of the numerous historical documents examined, that collective improvisation played a much greater role than had been assumed. For this reason, as Christian Meyer (1993) aptly notes, each of these documents which, in its singularity, probably reveals only one of the

¹ By ‘segment’ we mean here any section of a piece delimited by a cadential formula or a pause.

multiple possible realizations of a structure transmitted by the oral tradition – and not a “composition” – more generally raises the issue of the procedures by which writing was introduced into an oral tradition culture.

For this reason, musicologists can no longer simply study scores, which are merely a moment, a frozen instant in time of an eminently living and non-formalized practice, unlike ethnomusicologists working in the field on material that is constantly being renewed, and which those who practice these forms of music have most often not formalized. This questioning encourages us to examine with the greatest attention the relationships that medieval polyphony has maintained between writing and orality. To this end, a study of the forms of traditional polyphony that survive today can provide precious information.

Relations between Medieval and Georgian polyphony

While in Georgian polyphony, which was originally entirely oral, there is today some use of writing, with license to include improvisations, it is important to consider the relationships that it could have with medieval polyphony.

The analogies between Georgian polyphony and the polyphony of the Western Middle Ages have long been a subject of interest. As Frieder Zaminer rightly says:

“Historians who are familiar with medieval polyphony remain astonished by the similarity of certain musical forms found in the Caucasus” (Zaminer and Ziegler, 1993).

Susanne Ziegler adds:

“The analogies between medieval polyphony and the polyphony of some peoples of the Caucasus and even the hypothesis of a possible historical-genetic filiation between these two styles of music were formulated on several occasions during the 19th century. [...] Already at the beginning of the 20th century, musicologists had observed this resemblance when they heard for the first time in Central Europe recordings of polyphonic music from the Caucasus. [...] In a second phase – mostly in the 1930's –, Siegfried Nadel (1933) and Marius Schneider (1940) tried to provide a basis for this resemblance historically and genetically, while Nadel formulated, with many reservations, the hypothesis of a possible influence of one musical culture on the other” (*ibid.*).

For our analysis of Georgian polyphony, the idea at the outset was to draw on the methods used to analyze medieval polyphony. But it appeared that in the many studies of medieval polyphony, analyses dealing with the harmonic syntax are extremely rare. The fact is that there is no real method allowing for a systematic analysis of medieval polyphony.

It was therefore necessary to elaborate a new methodology, based on rigorous criteria and explicit procedures, which would be appropriate for the specific traits of Georgian polyphony. This is precisely the aspect that constituted the major difficulty in the analysis of Georgian polyphony. It was to overcome this difficulty that the method of reducing each piece to a *matrix* was developed.

Let us remember that many similarities in the Georgian and medieval repertoires were revealed as of the beginning of the 20th century. The common processes include drone polyphony, homophony, parallel, oblique, contrary and genuinely contrapuntal movement (by the latter we mean rhythmic independence of the parts).

Other characteristics involve respectively

- the use of modality,
- the cadences – internal and final – on so-called "perfect" intervals according to medieval theory, which systematically end on the unison, fifth or octave in both repertoires,
- the vertical conjunctions with alternation of dyads and triads,
- the melodic progression of the voices by conjunct degrees (step-wise),
- the importance of improvisation in these forms of music,
- the hypothesis by which medieval polyphony reflects the influence of a practice involving improvisation and that it is based, just as Georgian polyphony, on matrices, i.e.

stable syntactic structures – in other words, “on a generative system that can lead to different realizations of the same musical entity” (Meyer, 1993).

These similarities have never been studied systematically and in-depth and the rare attempts at comparison remain succinct. But beyond the value of modeling for the purpose of better understanding the « grammar » of Georgian polyphonic songs, this work could open up new perspectives for the analysis – or even the modeling – of vocal polyphony starting from the 13th century (École de Notre-Dame) until the beginning of the Renaissance. While these two repertoires seem to be very different – one of them is still practiced, based on oral transmission and improvisation, the other one is old, based exclusively on notation –, they share the existence of matrices in which the chords are separated by improvised ornamentations. The Georgian models confirm the approaches for the analysis of medieval music, encouraging us to do reductions of the notated pieces.

It could therefore be fertile to explore the obvious analogies between the oral polyphony of Georgia and the written polyphony of the Middle Ages through analysis and modeling. As Frieder Zaminer points out:

“ We cannot say today with certainty how our early polyphony sounded [in the Middle Ages]. What was set down in writing with the help of notation was in a sense the musical idea of the pieces that were performed. But as this idea [...] seems to us much too abstract, we are trying to give it a perceptible appearance. In doing this, the musical world of the Caucasus offers us fascinating stimulations” (Zaminer and Ziegler, 1993).

Once the proposed method had been shown to be operational for Georgian polyphony, it seemed promising to test medieval polyphonies with this new tool, and to see the extent to which research on the vocal polyphony of the Middle Ages could benefit from it.

To make the application of this method concrete, a mini-corpus of seven pieces from the Georgian liturgical repertoire and as many medieval liturgical pieces was constituted. This allowed us to identify points that are common to both repertoires and those that are different. All of the pieces selected are in three parts.

These Georgian chants are anonymous and are considered to be very old. They come from two monasteries – Gelati and Shemokhmedi. Six of the medieval chants, also anonymous, date from the twelfth to the fourteenth century; the seventh one is the motet *Ave Regina Caelorum* by Guillaume Dufay (fifteenth century)².

² The seven Georgian chants are:

- *Atskhovne upalo* (“O Lord, Save Thy People”), in ed. Nodar Mamisashvili, *Georgian Church Chant, Gelati school, vol. II (The Hymns to the Twelve Feasts of Our Lord...)* Tbilisi: Patriarchate of Georgia, 2006: 18-20.
- *Qovlisa dabadebulisa* (“Creator of All”), in Mamisashvili, *Georgian Church Chant*, vol. 2: 3-5.
- *Romelman hshv mtiebi* (“Thee, Who from Thy Virgin”), Mamisashvili, *Georgian Church Chant*, vol. 2: 11-12.
- *Metskhre galobis chasartavi* (“The Refrain to the IX Canticle”), Mamisashvili, *Georgian Church Chant*, vol. 2: 22.
- *Aghdgomasa shensa* (“To thy Resurrection”), in ed. Malkhaz Erkvanidze *Kartuli saeklesio da Salxino Sagaloblebi* (“Georgian Church and Feast Hymns”), *Gurian Chants*. Tbilisi: The Liturgical Chant Center of the Georgian Patriarchate, 2003: 3.
- *Dghes saghmrtoman madlman* (“Today by God’s Grace”), Erkvanidze, *Gurian Chants*: 29-30.
- *Dideba chvens shekrebas* (“Glory to our Gathering”), Erkvanidze, *Gurian Chants*: 39.

The seven medieval chants are:

- *Maria, Virgo Virginum*, in ed. Leo Schrade, *The Roman de Fauvel. The works of Philippe de Vitry. French cycles of the “Ordinarium Missae,”* vol. 1 (Monaco: L’Oiseau-Lyre, Collection ‘Polyphonic Music of the Fourteenth Century,’ 1974): 57.
- Anonymous, *Veri Floris Sub Figura*, C1, W1 2, 6, f. 15v.1, in Gordon Anderson ed., 1986, *Notre-Dame and Related Conductus*, volume 1, (Conductus-motets transmitted in conductus fascicules), Henryville, Ottawa and Binningen: Institute of mediaeval music, Collection ‘Collected works’ X,1.
- Anonymous, *Serena Virginum*, *ibid.*: A1, W1 2, 5, f. 13r.

Veri Floris Sub Figura

Metskhre Galobis Tchasartavi

Integral Transcription

The figure displays two musical transcriptions. The left transcription, 'Veri Floris Sub Figura', is in G major and 4/4 time, with a tempo of 60. It consists of four systems of music. The right transcription, 'Metskhre Galobis Tchasartavi', is in D major and 4/4 time, with a tempo of 69. It consists of four systems of music, including vocal lines with lyrics and a basso continuo line. Both transcriptions use a system of numbers and Roman numerals to indicate pitch and rhythm.

Figure 1. Integral transcriptions of the chants *Veri Floris Sub Figura* and *Metskhre galobis tchasavarti*.

- Anonymous, *Dic Christi Veritas*, *ibid.*: C3, W1 8, 4, f. 73r.
- Anonymous, *Conductus XV Mater patris et filia*, in ed. Higinio Anglés, *El Codex Musical de Las Huelgas: Música a veus dels segles XIII-XIV*, (Barcelona: Institut d'Estudis Catalans: Biblioteca de Catalunya n° 154, 1931), vol. 2.
- Anonyme, *Que nutrinus filios*, in ed. Leo Schrade, *The Roman de Fauvel. The works of Philippe de Vitry. French cycles of the Ordinarium Missae*, vol. 1 (Monaco: L'Oiseau-Lyre, Coll. 'Polyphonic Music of the Fourteenth Century,' 1/24, 1974).
- Guillaume Dufay, *Ave Regina Coelorum*, motet for 3 voices, Oxford Bodleian Library, MS. Canon. Misc. 213 (No. 129 Folio 62), edited by Rafael Ornes (1999).

Veri Floris Sub Figura

Metskhre Galobis Tchasartavi

Reduction

Figure 2 displays two sets of musical notation, each consisting of two systems of three staves (treble, alto, and bass clefs). The left set, titled "Veri Floris Sub Figura", is in 8/8 time with a tempo of quarter note = 60. The right set, titled "Metskhre Galobis Tchasartavi", is in 8/8 time with a tempo of quarter note = 70. Below the notation are numerical chord reductions using numbers 1-8 and Roman numerals I-VII.

Figure 2. Reduced transcriptions of the chants *Veri Floris Sub Figura* and *Metskhre galobis tchasavarti*.

The inventory of the Georgian chants shows 37 different vertical configurations, including 5 dyads and 32 triads³. The seven medieval chants have a total of 36, including 8 dyads and 33 triads⁴. This similarity between the number of different chords in each of the two sets is surprising in itself. But that is not all. Here, with 24 common chords⁵, their rate of appearance represents *two thirds* of the chords!⁶

Why Make Models?

For Georgian polyphony, the uncovering of the harmonic framework should allow us to find the matrix that is common to certain musical pieces *a priori* considered to be different. Conversely, it should also allow us to reveal musical differences between pieces, which up until now were considered to be identical – most often because of their words.

It thus allows for comparison of

- within the same piece, the variants between verses,

³ 1–3m–4–5–7; 2M/4–2m/4–3M/4–3m/4–3M/5–3m/5–3M/7M–3m/7m–4/5–4^m/5–4/6m–4^m/6M–4^m/7M–4/8–4^m/8–5/6M–5/6m–5/7M–5/7m–5/8–5/9M–5/10m–6M/7M–6M/8–6m/8–6M/9M–6M/10M–6M/10m–7m/8–7m/9–8/10M–8/12.

⁴ 1–2M–3M–3m–4–5–6M–8; 2M/3m–2M/5–3M/4–3m/4–3M/4^m–3M/5–3m/5–3M/6M–3M/8–3m/6m–3m/7m–4/5–4/6m–4^m/5–4^m/6M–4/7m–4/8–4^m/8–5/6M–5/6m–5/7M–5/7m–5/8–5/9M–6M/7m–6M/8–7M/8–8/12.

⁵ 1–3m–4–5; 3M/4–3m/4–3M/5–3m/5–3m/7m–4/5–4^m/5–4/6m–4^m/6M–4/8–4^m/8–5/6M–5/6m–5/7M–5/7m–5/8–5/9M–6M/8–6M/9M–8/12.

⁶ We note lastly that in both sets there are numerous successions of triads in which the fifth is systematically present.

- pieces from the same region belonging to the same repertoire (liturgical or secular, for example),
- pieces from different regions,
- liturgical chants from different monasteries and which have the same titles,
- two or several versions of the same liturgical piece from different monasteries and fulfilling an identical function in a ritual.

For medieval polyphony, the modeling could detect stylistic traits that might be specific to works of a given time period and/or a given place, or even attribute the paternity of certain works considered until now as anonymous, to known composers.

In all of these cases, the publication of modeled scores could contribute to reintroducing into each of them the practice of improvisation that was associated with it, but lost when it was set in writing.

Up until now, the analysis and modeling work has been done manually, which has limited the amount of data that could be processed and the number of questions that could be asked.

This research on Georgian polyphony, initially undertaken with a strictly ethnomusicological outlook, could thus contribute in a significant way to solving problems that involve musicology in general.

COMPUTATIONAL PERSPECTIVE

The analysis and modeling work described above has been done manually, which has limited the amount of data that could be processed and the number of questions that could be asked. The second approach uses a mathematical representation of songs as directed graphs (Scherbaum et al., 2015; 2016) to address the same questions as discussed above, e. g. to identify pillars and ornaments in a piece, from a computational perspective. Based on a digital representation of each musical score (in the musicXML language) each song is modelled as a discrete sequence of harmonic states, which we subsequently represent as a directed graph (Chartrand, 1985). This way, a song can (literally) be seen as a trajectory (which we call song path) on a map of harmonic states (which we refer to as chordscape). For a discussion of the theoretical background and for details of the method, the interested reader is referred to Scherbaum et al. (2016). Within the framework of this model, the analysis of chord progressions becomes simply the analysis of the shapes of song paths and the comparison of different scores becomes the comparison of images. Since songs with similar chord progression sequences will have similar song paths on the chordscape, image processing algorithms can be used very effectively to analyse the similarity of songs in terms of their harmonic structure and their temporal development. In conjunction with high-dimensional visualisation techniques such as Sammon's maps (Sammon, 1969), the similarity of all songs of a whole corpus can then be displayed as a two dimensional similarity map, as has been demonstrated by Scherbaum et al. (2016). This allows us to compare the harmonic chord progression structure of the Georgian and the Medieval Western European subset of our mini-corpus, respectively, as well as to visualize the effect of the classical musicological modeling approach, i.e. the influence of the removal of ornamental features of the songs. Fig. 3 shows the Sammon's maps for the chord progressions of all songs in our mini-corpus.

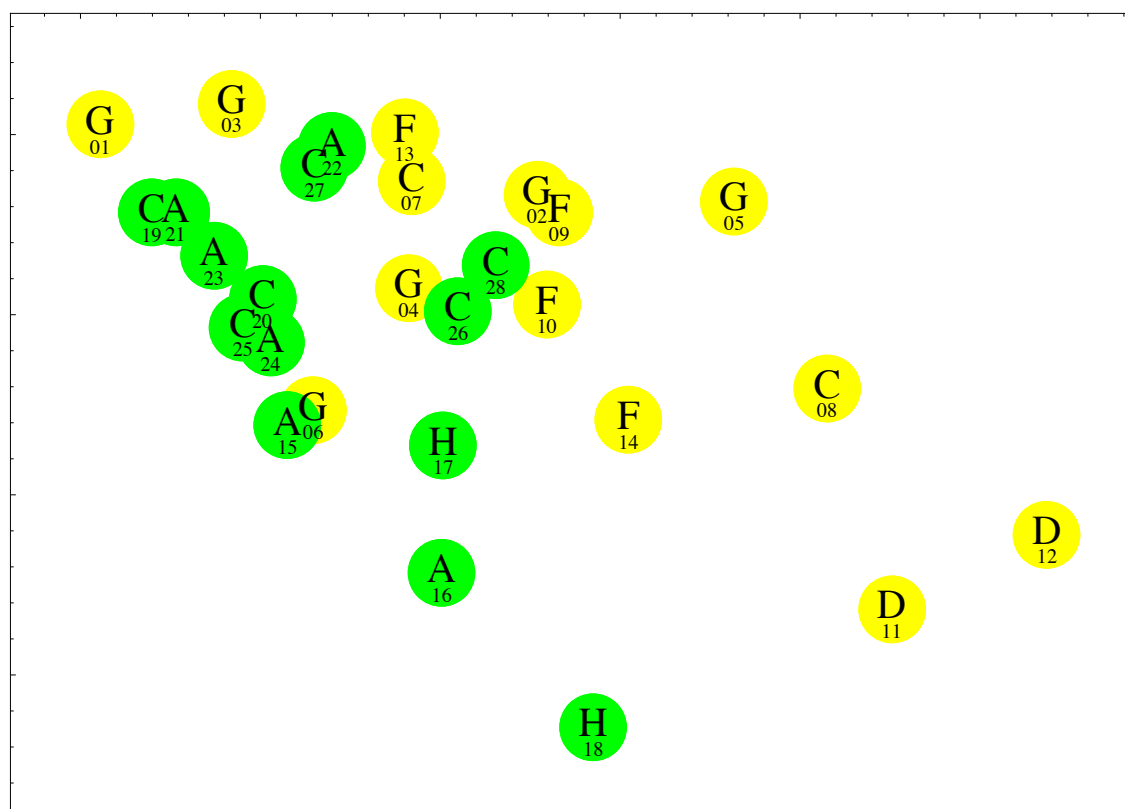


Figure 3. Sammon's map for the song paths images of all analysed songs.

#	ID	Name	Mode
1	MG01	Veri Floris (R)	G
2	MG02	Veri Floris	G
3	MG03	Serena Virginum (R)	G
4	MG04	Serena Virginum	G
5	MG05	Dic Christi Veritas (R)	G
6	MG06	Dic Christi Veritas	G
7	MG07	Mater patris et filia (R)	C
8	MG08	Mater patris et filia	C
9	MG09	Que Nutritos Filios (R)	F
10	MG10	Que Nutritos Filios	F
11	MG11	Ave Regina Coelorum (R)	D
12	MG12	Ave Regina Coelorum	D
13	MG13	Maria Virgo Virginum (R)	F
14	MG14	Maria Virgo Virginum	F
15	MG15	Qovlisa Dabadebulisa (R)	A
16	MG16	Qovlisa Dabadebulisa	A
17	MG17	Romelman (R)	H
18	MG18	Romelman	H
19	MG19	Metskhe Galobis (R)	C
20	MG20	Metskhe Galobis	C
21	MG21	Aghdgomasa Shensa (R)	A
22	MG22	Aghdgomasa Shensa	A
23	MG23	Dghres (R)	A
24	MG24	Dghres	A
25	MG25	Dideba (R)	C
26	MG26	Dideba	C
27	MG27	Atskhovne Upalo (R)	C
28	MG28	Atskhovne Upalo	C

Table 1. List of analysed scores. Scores 1 to 14 and 15 to 28, respectively, belong to the Medieval Western European and the Georgian songs subsets of the analysed mini-corpus. The label (R) in the name column refers to the reduced version of a song.

Each song is represented by a colored disk with two labels. The small number in the lower part of each disk refers to the song number in Table 1, while the letter in the center of each disk refers to the mode of the song. Songs numbered 1- 14 (color coded yellow) belong to the medieval Western European subset while song numbered 15-28 (color coded green) belong to the Georgian subset.

The two-dimensional mutual distances between the individual discs in Fig. 3, each representing a song, are reasonably good approximations of the mutual distances of the dissimilarity of the corresponding song paths. As can be seen in Table 1, the odd numbered songs correspond to the reduced version of the songs, in which all ornamental features have been removed as described in the previous chapter, while the even numbered songs refer to the original scores.

In order to analyse different scores which might cover a very different pitch range we have to first make them comparable. Here we have done this simply by using the bass voice of each chord as reference. This means that a three-voiced chord is defined by the two intervals of the middle and top voice, respectively, with respect to the bass voice, independent of the absolute pitch of the bass voice.

One can draw several conclusions from this map. First of all, one can see that the reduced and the original versions of the scores sometimes let the images become rather

dissimilar because the corresponding song disks move away from each other. However, the green labels (Georgian) and the yellow labels (Western European) stay apart. The closest it gets are songs number 6 and number 15. For the medieval subset, the mode of a song (cf. Arom and Vallejo, 2010; 2012) seems to matter somewhat because the individual modes are arranged systematically. Songs with mode D stay in the lower right while songs with mode G are distributed in the center and to the upper left. Songs in mode F stay somewhat together above the center. This would deserve further analysis but could not be done here due to the limited space available.

That observation that originals and reductions are sometimes close, sometimes not is actually not surprising since reduction can change the chord progression sequence and hence the song path quite considerably.

Fig. 4 shows the matrix of the most frequently used chords in the complete collection used in both subsets (except for the rests, which are only used in the Georgian subset). Please note that since the individual scores are vertically sorted according to their mode types, the uppermost 7 songs correspond to the Georgian while the lowermost 7 songs belong to the Medieval Western European subset of the songs, respectively.

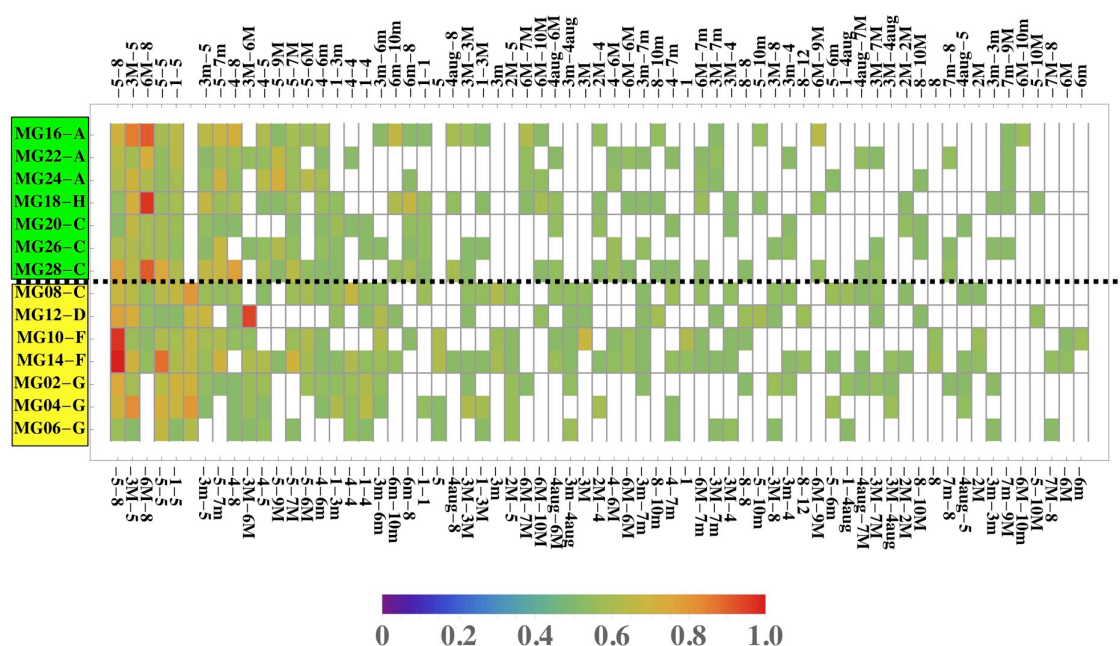


Figure 4. Matrix of most frequently used chords (sorted from left to right) in all songs. Chord labels refer to the intervals with respect to the bass voice.

Fig. 4, does not show any striking differences between the Georgian and the Western European subset in terms of the chords used. In fact, the variability of chords used within each subset is larger than between the subsets.

This is no longer the case for the chord **transitions** (Fig. 5). One can see that some transitions are used in both subsets, but some transitions only in one of the subsets. This suggests that the rules for combining the chords (the syntax) in the Georgian and the Western European subsets seems to be different, even if the chords used are similar.

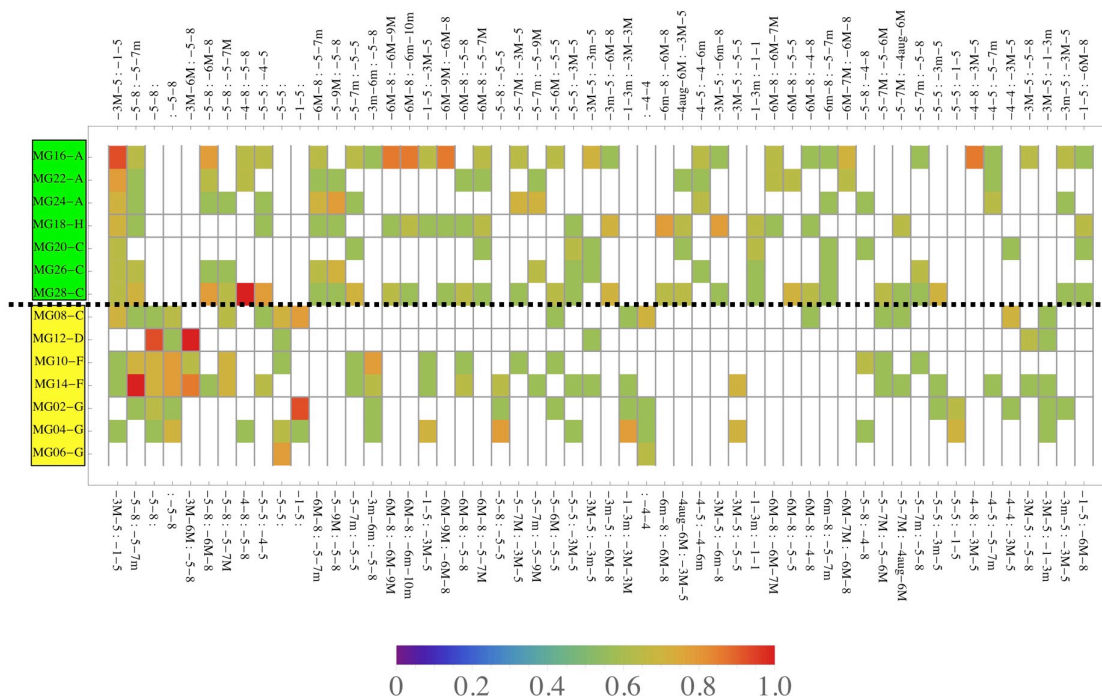


Figure 5. Matrix of most frequently used chord transitions (sorted from left to right) in all songs. Chord labels refer to the interval with respect to the bass voice.

DISCUSSION

In the present paper, we have investigated a small corpus of three-voiced Georgian and Medieval songs regarding the used chord inventories and the syntax of the chord progressions from two conceptually very different perspectives.

In the first, the **classical perspective**, the modelling process employs musicological expert knowledge during all stages of the modelling process. The resulting model is rather easy to interpret because the goal of the modelling process is usually clearly defined (e. g. „to obtain the fundamental chord progression structure of a song“). Interpretation of the model then follows naturally.

In the second, the **computational perspective**, the modelling process starts from data processing („let the data speak“). Musicological expert knowledge is not used in the initial phase. The resulting models (images) are not always easy to interpret for musicologists. The modeling can nicely be used in an exploratory sense and on large corpora to find representations which can lead to the discovery of new knowledge or better questions.

As a concrete result of this originally explorative study, we can state that, as far as the small collection of investigated songs is concerned, the syntactical rules (grammar) used for the Western European Medieval and the Georgian subset of our collection are different.

In addition, this short and because of the limited space admittedly superficial discussion may already suffice to illustrate the potential of combining classical and computational approaches to address ethnomusicological problems.

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