Graphical comparative analysis of the harmonic structure of the Akhobadze corpus of Svan songs

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1. INTRODUCTION

The present paper proposes a computational approach to the comparative analysis and visualization of the harmonic structure of three-voiced vocal music. The dataset which we have been using in this study is the same as in Scherbaum et al. (2015), a corpus of polyphonic songs from Svaneti (Akhobadze, 1957). Similar to our earlier work, a song is treated as a discrete temporal process in which harmonic or melodic states change according to unknown rules which are implicitely contained in the song itself. In contrast to our prior study, however, there are no assumptions regarding the probabilistic or deterministic nature of these rules.

2. METHODOLOGICAL FRAMEWORK

In the preprocessing phase of the analysis, which is described in detail in Scherbaum et al. (2015), each score (e. g. Fig. 1) was manually transfered into a computer readable XML format and checked against a set of usability criteria (e.g. only three voiced songs of certain length were considered) and converted into a temporal sequence of harmonic states (Fig. 2).

This representation completely captures the inventory and progression of harmonic states in the song, but in this form (as list of lists of labels and numbers) not very convenient for human analysis. In order to facilitate the musicological interpretation we decided to first transform the harmonic state representation above into mode degrees following the approach of Arom and Vallejo (2008), which was implemented as an automatic procedure described in Scherbaum et al. (2015). It turned out that 99% of the usable songs where in mode La (75%), Sol (21%), and Re (3%). These modes differ only in the sizes of their 3rds and 6ths being sung as minor or major. Based on the long standing practical experience of one of us (FK) with Svan singing, but also based on the observation of recent recordings of authentic Svan singers (Scherbaum, 2016) which suggest that 3rds and 6ths in traditional Svan music are neither sung as minor nor as major intervals, it was concluded that the separation of the Akhobadze corpus into different modes is not sufficiently supported by the data. For the subsequent analysis it was therefore provisionally assumed that all songs belong to a single 7-step mode in which the distinction between minor and major intervals is dropped, but for which the particular scale does not have to be specified.

In order to facilitate the visualization, we choose a subscript-superscript notation for the harmonic states in which roman numerals indicate the pitch of the bass note of a chord (or simply the pitch of the note in case of a monophonic part) expressed as mode degree. The intervals formed by the middle and top voice against the bass are given as subscript and superscripts, respectively. The whole chord (harmonic state) is put into parenthesis and an outer superscript denotes its duration. For example, the first three-voice chord of the song {{E4, D4, A3}, 2} translates into $(VII_4^5)^{1/2}$ with the bass not A3 corresponding to the mode degree VII (the finalis is on B3) and the notes D4 (middle voice) and E4 (top voice) to a 4th and a 5th above it. The duration is 2 quarter note beats which corresponds to a 1/2 note, as indicated by the outer superscript.

Therefore, for the subsequent analysis we concentrate on the 72 core songs which passed all the quality criteria. Since for now we are focussing on the harmonic (and not the temporal) structure of the songs, we drop the duration information from further considerations. For each song this leaves us with a simple and easy-to-read sequence of chord progressions (Fig. 3). The generation of these chord progression tables concludes the preprocessing and provides the basis for the subsequent analysis.

In order to obtain a first idea of the harmonic structure of the songs in the analysed corpus, we started out with a visual syntactic analysis of a few selected songs in the course of which we manually segmented and rearranged the chords in such a way that all reiterative sequences of two or more "states" are easily identified by their horizontal position in the rearranged table. For the song "Tamar mepla" this paradigmatic form is shown in Fig. 4.

Fig. 4 provides an effective way to visualise and analyse the harmonic organization of the song. The tabular form of the grid is well suited for the analysis of individual songs. However, it becomes less convenient for the joint analysis of several songs since in general they will be based on at least partially different chords and the corresponding tables become quite difficult if not impossible to interpret. Therefore, for the following analysis we choose a different representation, namely as as so-called directed graphs (e. g. Chartrand, 1985), which for individual songs is nicely related to the tabular form of Fig. 5, but which can be also be used for the joint analysis of a set of songs and which in addition offer a number of other convenient features.

3. REPRESENTATION OF SONGS AS DIRECTED GRAPHS

As can be seen from Fig. 4, the complete song is based on the following chords (here ordered in the sequence in which they first appear in the song): $\{ VII_4^5, VII_4^4, VII_4^6, V_5^7, VI_3^5, VII_1^3, I_1^5, VII_3^5, VI_5^5, V, I_1^3, I_2^4, I_3^5, III, VII_2^4, I_1^1 \}$ with III and V being single notes on the mode degree III and V, respectively. If we plot the set of these chords as a set of verteces along a circle as done in Fig. 5 a), we can represent a single harmonic progression in the song as an edge connecting one vertex with another. Mathematically speaking, this corresponds to the representation of the song as a directed graph G=={V,E} (Chartrand, 1985) which consists of a set of vertices V (representing the harmonic states of the song) and a set of edges E which represent the chord progressions.

Fig. 5 b) illustrates for example the progression from the beginning of the three-voice part of the song (chord index 4 in Fig. 4) to the first chord in the second row of Fig. 4 (chord index 14). Each step is indicated by an arrow with the last progression in the sequence shown in green. Fig. 5 c), d), and e) show the further progression to chord indices 15, 16, and 22, respectively. Fig. 5 f) shows the graph for the complete song. The temporal harmonic development, but also the relationship of the graph representation of the song to the paradigmatic table form of Fig. 4 is better seen if we add a vertical time axis to Fig. 5 as illustrated in Fig. 6. Again, the chord progression is shown as a path over the chord verteces oriented along a circle. However, with each step in the sequence the path moves upwards by a small amount. This way, it becomes immediately obvious which chords are visited more than once and how often. In other words, the repetition of chord segments, which in Fig. 4 can be deduced from the row structure of the table becomes now immediately visible in the vertical structure of Fig. 6.

Although Fig. 6 nicely illustrates the temporal development and the organization of the chord progression as a sequence of loops and straight segments for an individual song, its complexity (as 3D plot) make it less useful for routine analysis and in particular for the comparison of songs which seems to be better done in a 2D representation. However, in order to simplify the display but still keep the information regarding which of the chords are used more than once and how often, we change the thickness of the arrows according to the number of times a particular segment has been used within the song as shown in Fig. 7.

Although Fig. 7 already nicely condenses the information regarding the harmonic structure of the song "Tamar Mepla", the complexity of the graph can still be reduced by reorganising the vertex positions such that the number of edge crossings is minimized. Technically speaking this corresponds to the so-called "Tutte layout" of the graph which is shown in Fig. 8a. Fig. 8 shows the harmonic structure of the song "Tamar Mepla" in a very efficient graphical way in which both the proximity of the verteces and the thickness of the edges carry useful information. The proximity of the verteces reflects a similarity in the frequency of the chords being used. This is further illustrated in Fig. 8b) which shows the number of times the corresponding chord is used in the song "Tamar Mepla". It can be seen for example that the most common chord is VI_3^5 which is used 10 times, followed in frequency by the chords VII_4^5 (the starting chord), V_5^7, and VII_1^3 each of which are used 8 times. From the edge thicknesses in Fig. 8a) on the other hand, one can see for example that the sequence V_5^7, VI_3^5 VII_1^3 is the most often used chord progression in the whole song.

3.1 Two songs

Within the graphical framework, an individual song is simply a “path” (called “song path”) in a "landscape of chords" which will be referred to as "chordscape". The thickness of the individual segments of a song path reflects how often the particular segment is "travelled". The interpretation of songs as directed graphs might require some training on the side of a musicologist but its advantages become obvious in the context of analysing a whole set of songs together. Naturally, the concept of song paths is easily expanded to a larger group of songs by simply adding new chords (as vertices) to the chordscape and recalculating their optimum positions so that the number of path crossings of all song paths is minimized. This is illustrated in Fig. 9 for the combination of two songs.

Fig. 9 nicely shows which of the chords are jointly used by both songs and which of the chords are used exclusively by one or the other. On closer inspection it also reveals interesting details for example that although both the chords VI_3^5 and VII_3^5 are used in both songs, they are used in different ways. In song number 5 (top panel) there are direct chord progressions between the two chords while in song number 9 their are not. We don´t want to elaborate on this here but just give this as an example of the kind of features which can be derived from the comparison of song paths simply by visual inspection. Again it is worth noting that because the position of the verteces is calculated such that the number of path crossing is minimized, the proximity of chord vertices reflects a similarity in the number of times the chord is used in the complete set of songs analysed (here two). This becomes even more relevant for the complete corpus as will be shown below.

3.2 Whole corpus

As already discussed and illustrated above, the representation of songs as directed graphs and their display as song paths in a chordscape offers interesting ways of graphically analysing the harmonic organization of songs and their relationships. Due to the size restrictions for this paper, however, we can only scratch the surface of the potential of this framework and highlight some selected features. If for example we display all song paths in a single graph on the full chordscape of the whole corpus we obtain Fig. 10. Again, the positions of the vertices was calculated such that the number of total path crossings is minimized. From their position in the outer locations of the chordscape, one can immediately identify those chords which are less often used (maybe even only once) while the most used chords in the corpus VII_3^5: 529 times; VI_3^5: 332 times, I_3^5: 322 times, VII_1^3: 182 times, VII_4^5: 156 times, I_4^5: 152 times, I_1^1: 144 times, I_4^6: 112 times, I_1^3: 107 times) are found in the center of the cluster.

One can now also quantitatively calculate the relationship of songs in terms of their harmonic organization. An elegant way to do this is offered by a so-called Sammon´s map (Sammon, 1969) as shown in Fig. 11.

Without going into technical details here, it suffices to say that the points on the Sammon´s map are calculated in such a way that the two-dimensional mutual distances between the individual points, each representing a song, are reasonably good approximations of the mutual distances of the dissimilarity of the corresponding song paths. To illustrate this even further, Fig. 12 shows four examples for songs which differ in their mutual dissimilarities considerably.

The two songs in the top panel of Fig. 12 (AKH68 and AKH76) have very different song paths and hence harmonic structures which is reflected in the Sammon´s map in Fig. 11 by their opposite locations while the songs AKH30 and AKH47 which are closely located in Fig. 11 show much more similarity in their harmonic structures.

There is a multitude of "statistical" questions musicologists who are interested in the analysis of the harmonic organisation of songs might be interested in but are difficult to address quantitatively with classical approaches e. g. those of Arom and Vallejo (2008) at least for large corpi. For the present approach the corpus size, however, is no longer a real obstacle. Questions such as "What is the longest sequence of chord progressions in the whole corpus which occurs in more than one song?" or "What is the most common sequence of 6 chords which occurs in more than one song and where?" or many similar questions can easily be answered with the tools in our Mathematica based toolbox for score analysis. Just to illustrate this point, the answer to the first question is the sequence $\{VII_3^5, VI_3^5, VII_4^5, VII_3^5, VI_3^5, VII_1^3, I_1^1\}$ and it occurs in songs number 45 and 46 while the answer to the second question is the sequence $\{VI_3^5, VI_5^5, VII_4^5, VII_3^5, VII_1^3, I_1^1\}$ which occurs 9 times in songs number 13 and 22, all derived with a few lines of Mathematica code.

4. CONCLUSIONS

The representation of songs as directed graphs allows the quantitative analysis of the harmonic organization of individual songs in a graphical, transparent and reproducible way. It also provides a framework to quantitatively compare the similarity of songs in a whole corpus e. g. by using high-dimensional visualisation techniques such as Sammon´s maps. The resulting neighborhood relations from the latter analysis can be displayed in ways which can be used for further musicological studies even by non-mathematically inclined analysts. Although the analysis was concentrated on the Akhobadze corpus of Svan music, the approach can easily be applied to music from other regions as well and score analysis in general. Despite it being primarily intended as a methodological study, the present analysis raised a number of questions regarding the transcription process and revealed a number of interesting findings regarding the modal structure and the harmonic organisation of the songs in this corpus.

5. REFERENCES

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Figure 1. Musical score of the song "Tamar mepla" (Akhobadze song number 9). For the analysis as three-voiced song, the quarter note of A3 in the chord in measure three is ignored and F#3 taken as bass note

{{{None, B3, None}, 1}, {{None, C#4, None}, 1/ 2}, {{None, D4, None}, 3/2}, {{E4, D4, A3}, 2}, {{D4, D4, A3}, 1/2}, {{F#4, D4, A3}, 1/2}, {{E4, C#4, F#3}, 1}, {{D4, B3, G3}, 1/2}, {{C#4, A3, A3}, 3/2}, {{F#4, B3, B3},1}, {{E4, C#4, A3}, 1/2}, {{D4, D4, G3"}, 1}, {{F#4, None, None}, 1/2}}

Figure 2. Harmonic state representation of the first 12 beats of the song "Tamar mepla" (Akhobadze song number 9). Each state contains a list of pitches (top, middle, bass voice} followed by a duration in terms of quarter note beats.

AKH09 Tamar Mepla

	I	II	III	VII^{5}_{4}	VII^{4}_{4}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	I^{5}_{1}	VII^{5}_{3}	VI^{5}_{5}	V	VII^{5}_{4}	VII^{4}_{4}
IDX	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	I^{3}_{1}	I^{4}_{2}	I^{5}_{3}	III	VII^{5}_{4}	VII^{5}_{3}	VII^{4}_{2}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	I^{5}_{1}
IDX	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	VII^{5}_{3}	VI^{5}_{5}	VI^{5}_{3}	VII^{5}_{4}	VII^{4}_{4}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	VII^{5}_{4}	VII^{4}_{4}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}
IDX	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
	I^{5}_{1}	VII^{5}_{3}	VI^{5}_{5}	V	VII^{5}_{4}	VII^{4}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	I^{3}_{1}	I^{4}_{2}	I^{5}_{3}	III	VII^{5}_{4}	VII^{5}_{3}
IDX	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	VII^{4}_{2}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}	I^{5}_{1}	VII^{5}_{3}	VI^{5}_{5}	VI^{5}_{3}	VII^{5}_{4}	VII^{4}_{4}	VII^{6}_{4}	V^{7}_{5}	VI^{5}_{3}	VII^{3}_{1}
IDX	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
	I^{1}_{1}														
IDX	76														

Figure 3. Progression of harmonic patterns in the song "Tamar mepla" expressed in degrees according to the assumed Svan tuning as described above. Displayed are the mode degrees of the bass note and the generic intervals on top of it. The numbers in the rows below the chords correspond to the chord index, numbered from 1 to the maximum number of chords/states in the song.

AKH09 Tamar Mepla

	VII^5_4			VII^4_4	VII^6_4	V^7_5	VI^5_3	VII^3_1		I^5_1	VII^5_3	VI^5_5					V	
IDX	*4*			*5*	*6*	*7*	*8*	*9*		*10*	*11*	*12*					*13*	
	VII^5_4			VII^4_4		V^7_5	VI^5_3	VII^3_1	I^3_1					I^4_2	I^5_3	III		
IDX	*14*			*15*		*16*	*17*	*18*	*19*					*20*	*21*	*22*		
	VII^5_4	VII^5_3	VII^4_2		VII^6_4	V^7_5	VI^5_3	VII^3_1		I^5_1	VII^5_3	VI^5_5	VI^5_3					
IDX	*23*	*24*	*25*		*26*	*27*	*28*	*29*		*30*	*31*	*32*	*33*					
	VII^5_4			VII^4_4	VII^6_4	V^7_5	VI^5_3	VII^3_1										
IDX	*34*			*35*	*36*	*37*	*38*	*39*										
	VII^5_4			VII^4_4	VII^6_4	V^7_5	VI^5_3	VII^3_1		I^5_1	VII^5_3	VI^5_5					V	
IDX	*40*			*41*	*42*	*43*	*44*	*45*		*46*	*47*	*48*					*49*	
	VII^5_4			VII^4_4		V^7_5	VI^5_3	VII^3_1	I^3_1					I^4_2	I^5_3	III		
IDX	*50*			*51*		*52*	*53*	*54*	*55*					*56*	*57*	*58*		
	VII^5_4	VII^5_3	VII^4_2		VII^6_4	V^7_5	VI^5_3	VII^3_1		I^5_1	VII^5_3	VI^5_5	VI^5_3					
IDX	*59*	*60*	*61*		*62*	*63*	*64*	*65*		*66*	*67*	*68*	*69*					
	VII^5_4			VII^4_4	VII^6_4	V^7_5	VI^5_3	VII^3_1										I^1_1
IDX	*70*			*71*	*72*	*73*	*74*	*75*										*76*

Figure 4. Paradigmatic form of the progression of harmonic patterns in the song "Tamar mepla" expressed in degrees according to the assumed Svan tuning as described above. Displayed are the mode degrees of the bass note and the intervals on top of it. The numbers in the rows below the chords correspond to the chord index, numbered from 1 to the maximum number of chords/states in the song.

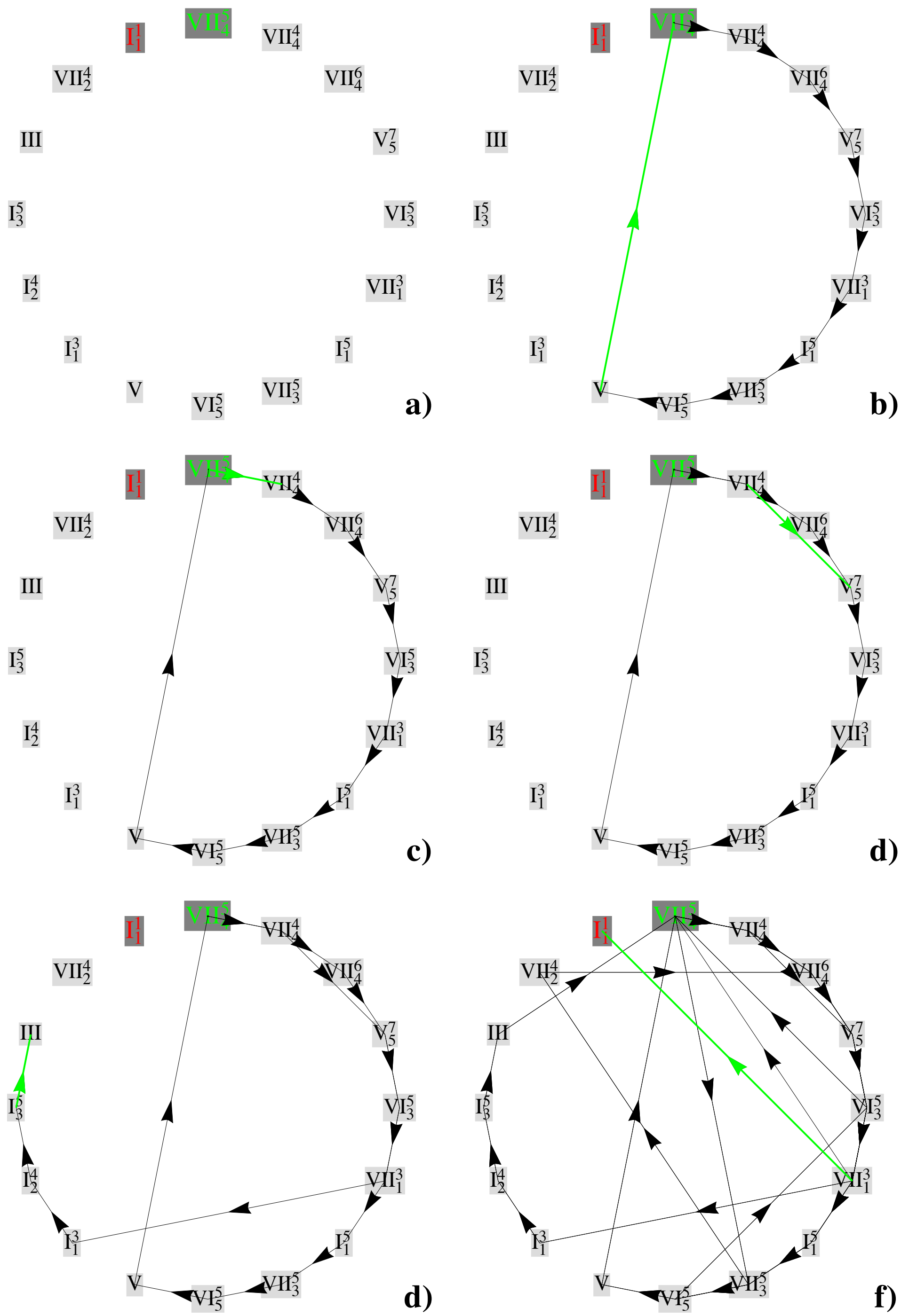


Figure 5. The song "Tamar Mepla" represented as a directed graph. Each chord in the song is represented by a labeled vertex with the first and the last chord in the song shown in green and red, respectively. A progression from one harmonic state to another is shown as an arrow, with the last arrow in a sequence of progressions shown in green.

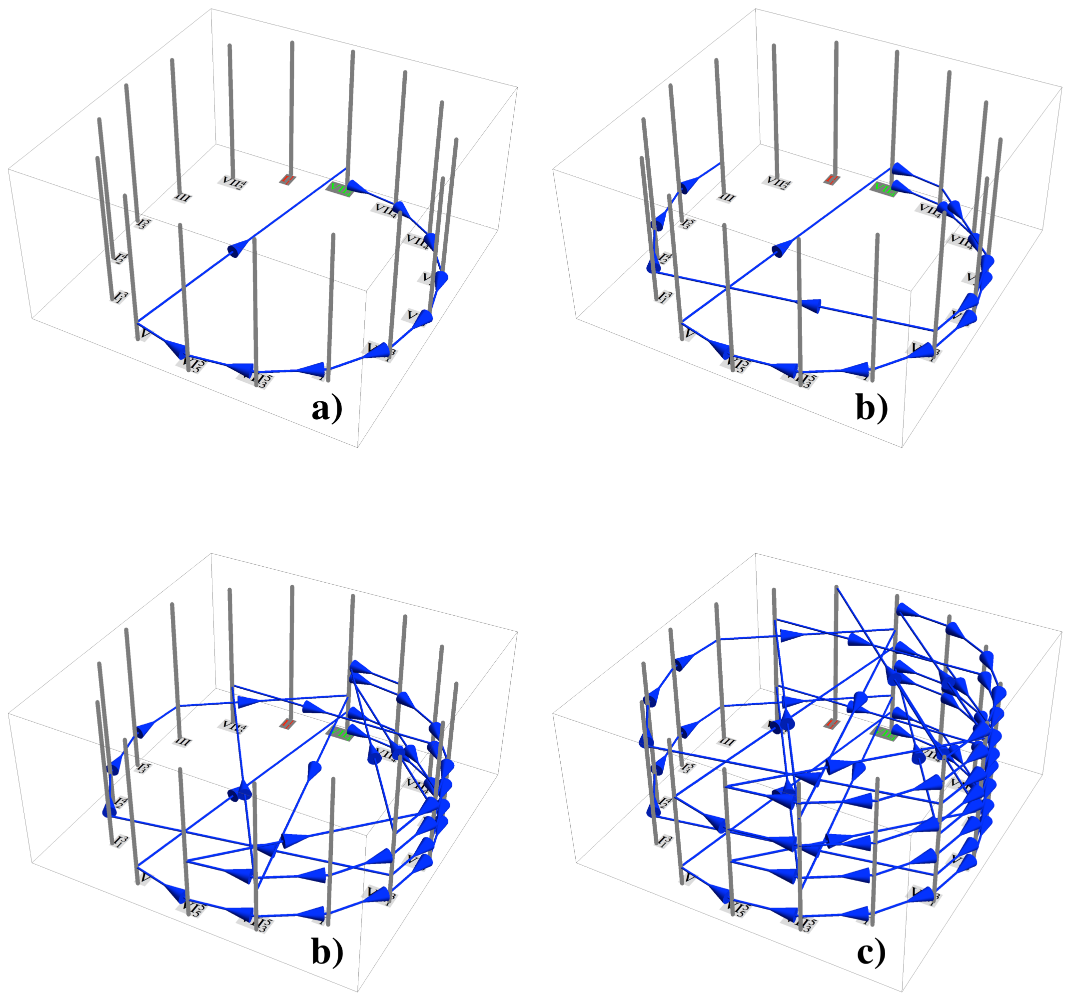


Figure 6. Chord progression in the song "Tamar Mepla" shown as 3D directed graph. The horizontal position of the chord verteces is the same as in Fig. 6. The vertical axis corresponds to time. Fig. 6 a-c show snapshots of different temporal stages of the song.

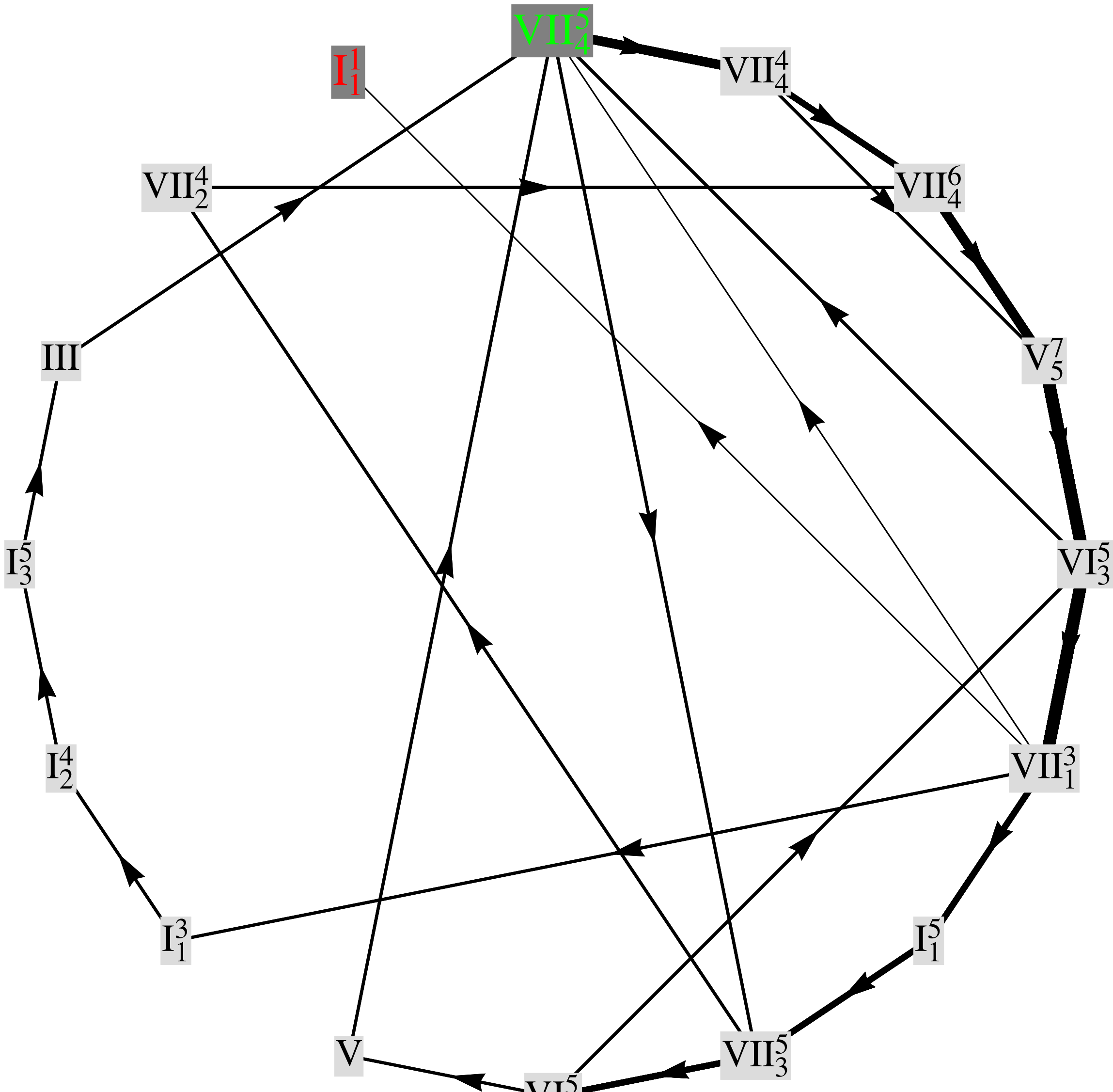


Figure 7. Graph representation of the song "Tamar Mepla" in which the line thickness of each edge indicates how many times the corresponding chord progression is used in the song.

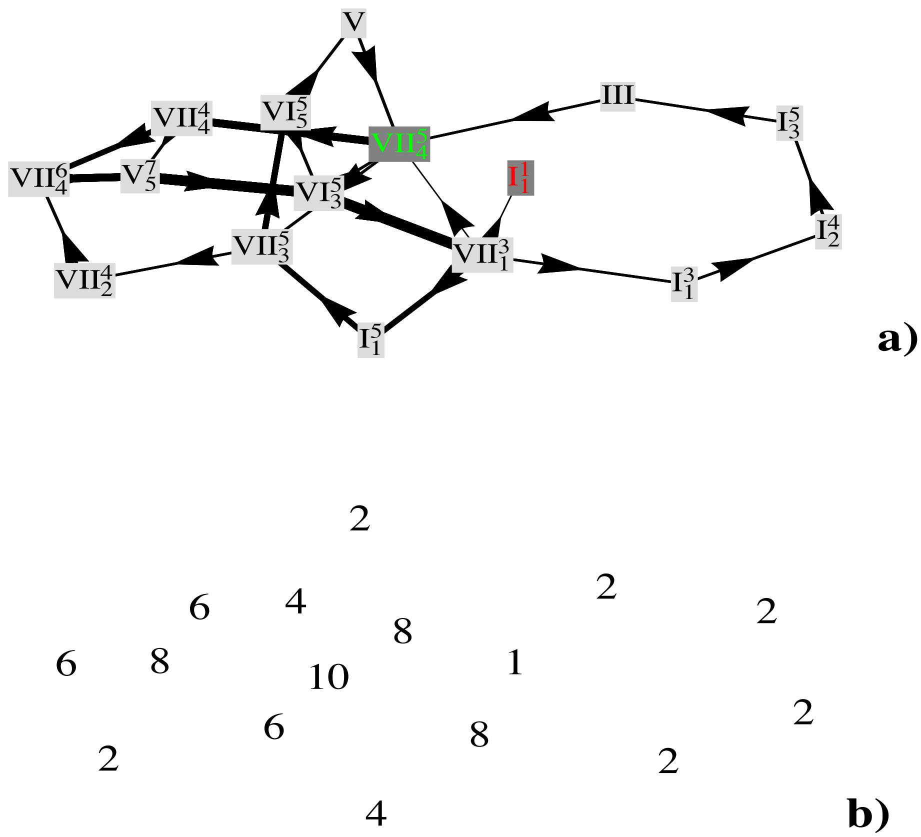
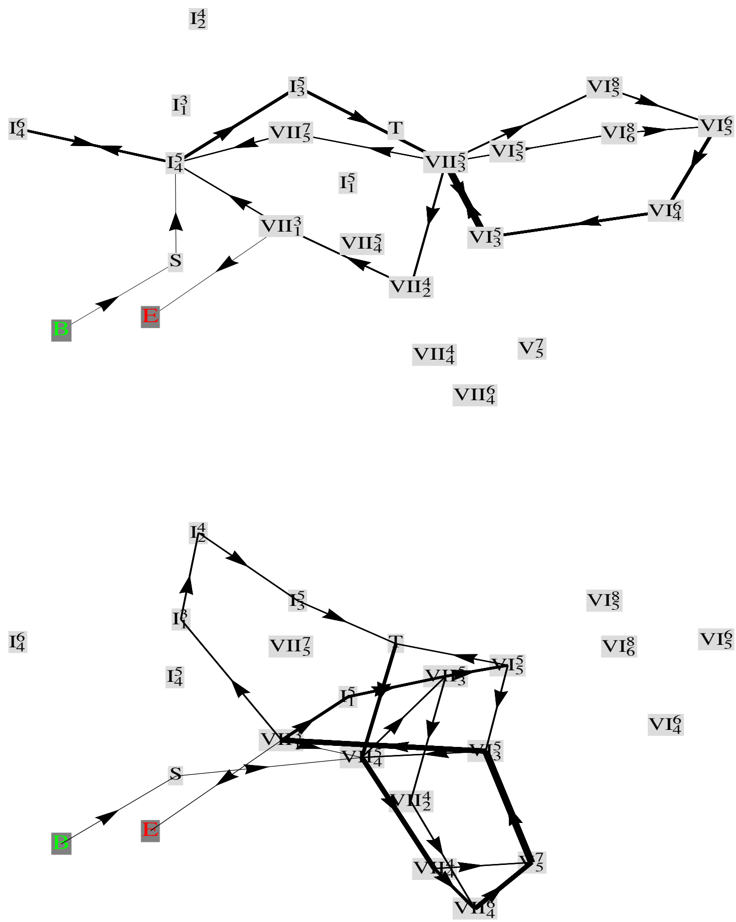


Figure 8. Reorganizing the vertex positions in the graph in Fig. 7 such that the number of edge crossings of the graph is minimized. Fig. 8. a) shows the graph while Fig. 8. b) shows the number of times the corresponding chord is used in the song.



a)

b)

Figure 9. Chordscape and song paths for the combination of two songs. Fig. 9a shows the song path for Akhobadze song number 5 "Mgzavruli" while Fig. 9b shows the song path for Akhobadze song number 9 "Tamar Mepla". The begin and end of both songs are shown by the generic labels "B" in green and "E" in red, respectively. The line thickness indicates how many times a particular segment is used in the song.

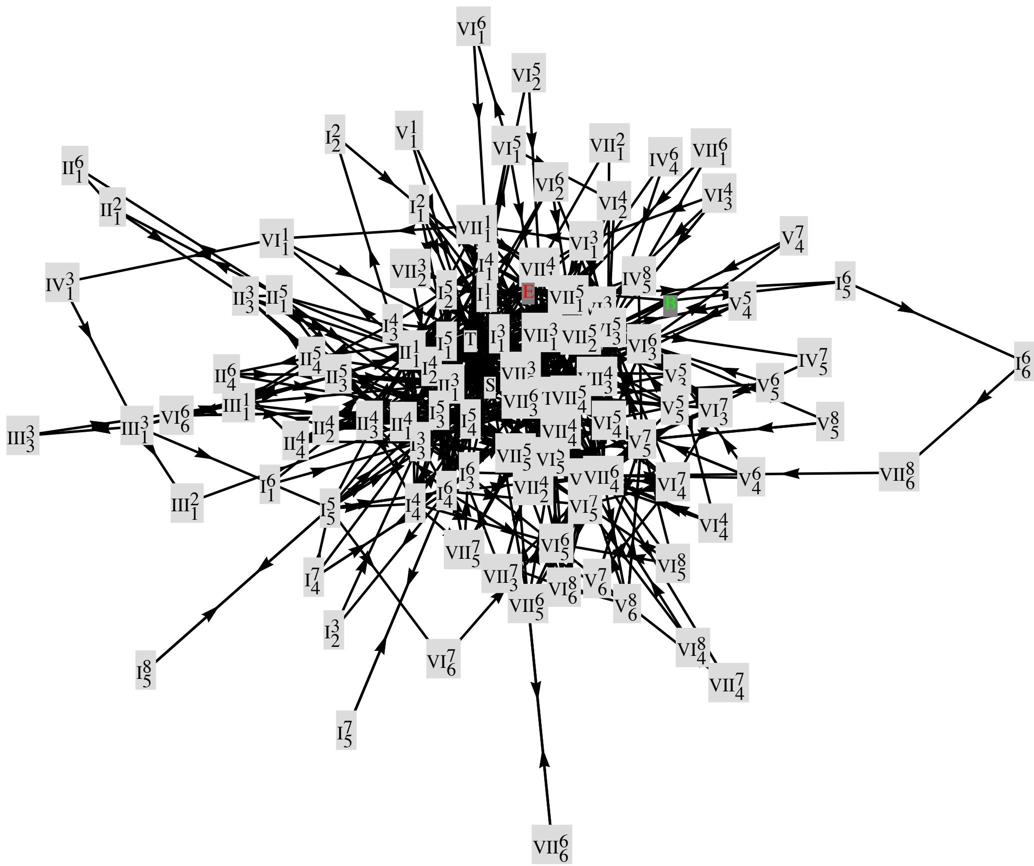


Figure 10. Joint song paths for all songs in the corpus plotted on top of the chordscape for the complete corpus. The complete chordscape consists of a total of 102 different chords and 4 generic labels indicating the begin (B), end (E), non-three voice start of a song (S), and non-three voice transition segments within songs (T).

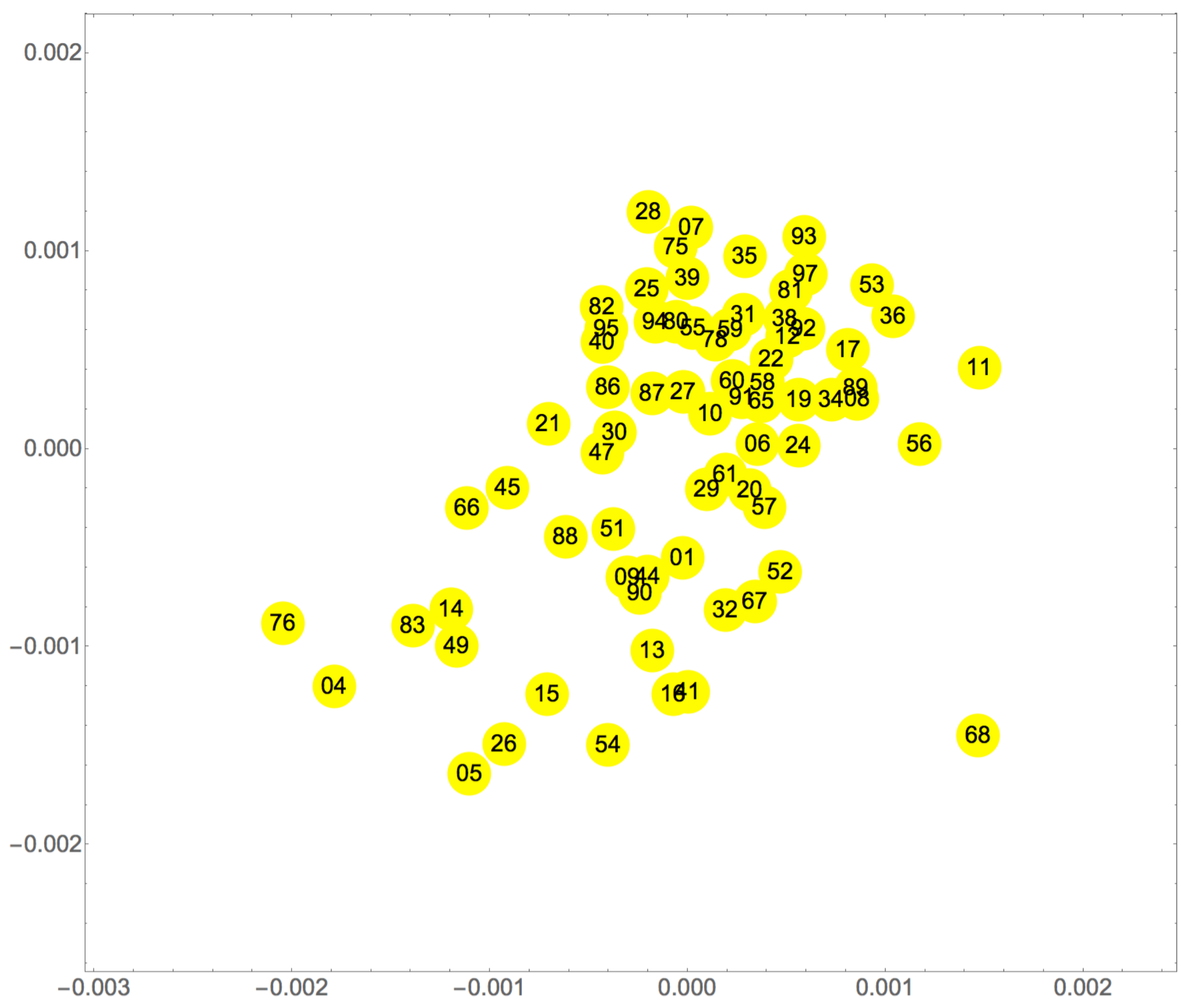


Figure 11. Sammon´s map for the song paths images of all analysed songs. The two-dimensional mutual distances between the individual points, each representing a song, are reasonably good approximations of the mutual distances of the dissimilarity of the corresponding song paths.

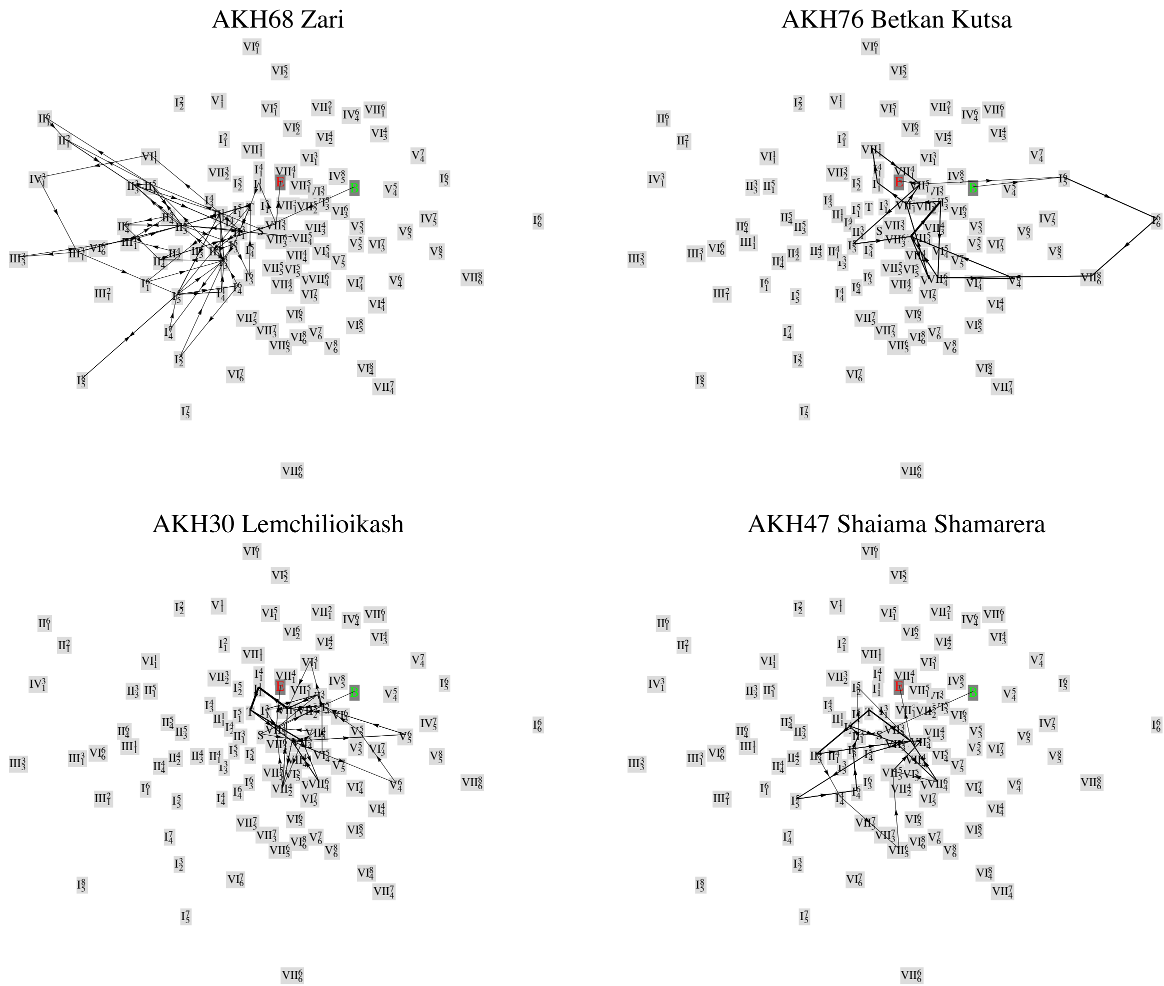


Figure 12. Examples of song path images for songs which have very different (top panel) and rather close (bottom panel) harmonic structures.