Optimal Energy Taxation in Cities

by

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March 2016, Revised August 2017

Abstract

This paper presents the first investigation of the effects of optimal energy taxation in an urban spatial setting, where emissions are produced both by residences and commuting. When levying an optimal direct tax on energy or carbon use is not feasible, the analysis shows that exactly the same adjustments in resource allocation can be generated by the combination of a land tax, a housing tax, and a commuting tax. We then analyze the effects of these taxes on urban spatial structure, showing that they reduce the extent of commuting and the level of housing consumption while increasing building heights, generating a more-compact city with a lower level of emissions per capita.

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1. Introduction

In step with growing concerns about the impact of global warning, urban research has increasingly focused on the energy consumption of cities. This research reflects the recognition that residential and commercial land-uses are important generators of greenhouse gas (GHG) and local emissions along with the transportation and industrial sectors. Their importance is seen in Table 1, which shows energy use by sector, with emissions from electricity generation "distributed" according to the final users of the electricity. As can be seen, when their electricity use is taken into account via the distribution method, the residential and commercial sectors each account for an appreciable 16.9% of total emissions, with their 33.8% total exceeding the shares of industry and transportation. Therefore, economic analysis of policies designed to control emissions should ideally include these two real-estate sectors in its focus along with other sources.

In advancing this goal, some researchers have studied the relationship between a building's energy use and its structural characteristics, with notable contributions by Costa and Kahn (2011), Chong (2012) (who also draws a link to climate), and Kahn, Kok and Quigley (2014). Using a hedonic approach, Eichholtz, Kok and Quigley (2010) ask whether the market values green buildings, finding that energy-efficient commercial structures indeed command higher rents. Glaeser and Kahn (2010) extend the focus beyond residential energy use to include emissions from driving and public transit, generating a ranking of US cities according to their overall carbon footprints. Zheng, Wang, Glaeser and Kahn (2011) extend this approach to Chinese cities.

In parallel with these empirical efforts, other researchers have imbedded energy usage into the familiar monocentric-city model of urban economics, with the ultimate goal of appraising the effect of urban policy interventions on emissions. These studies, most of which rely on numerical simulations of realistically calibrated urban models, include Larson, Liu and Yezer's (2012) study evaluating the energy-use impacts of higher gasoline taxes, better vehicle fuel efficiency, urban greenbelts, and housing density restrictions. Larson and Yezer (2015) ask how such policy impacts vary with city size, while Borck (2016) explores the effect of building-height limits on urban GHG emissions. Tscharaktschiew and Hirte (2010) study the impact of emission taxes and congestion on emissions from commuting. In contrast to these papers, Schindler, Caruso and Picard (2017) develop a monocentric-city model with spatially heterogeneous pollution, which depends on the volume of CBD-bound traffic passing a given location. Their analysis characterizes the socially optimal city and derives the tax structure needed to support it (a lump sum tax that rises with distance is required).[1]

Although this second group of studies has greatly increased our understanding of the links between urban spatial structure, energy use, and emissions, an important question remains unanswered. In particular, no study has analyzed socially optimal urban form when both housing and commute trips generate emissions. While Schindler et al. (2017) partly fill this gap with their model of location-specific auto pollution, the present paper goes farther. We add energy use and both GHG and local emissions to the housing sector of the standard urban model, doing so in a novel and realistic fashion, while also recognizing the emissions from commuting. With both types of emissions assumed to reduce consumer utilities, the analysis then develops the conditions that characterize the optimal city, which embody a trade-off between the environmental gains from lower emissions and the losses from achieving them.

The form of these optimality conditions reveals that real estate taxes of a particular form along with a commuting (or gasoline) tax are needed to generate the optimum. Within the context of the model, the resulting allocations are in fact the same ones that would be generated by a carbon tax (or generalized emissions tax) on gasoline and on the fuels used to produce residential energy. Despite this equivalence, the analysis focuses on the real estate and gasoline taxes themselves rather than on the underlying carbon tax, partly because doing so facilitates second-best analysis, where one or more of these taxes is set at zero. For example, zero real estate taxes can be imposed, forcing the gasoline tax to address emissions from both residences and commuting, an exercise that is impossible under a carbon tax.[2] With this theoretical foundation, numerical simulation analysis then derives the changes in urban form that follow from imposition of the optimal taxes. The simulation results thus show how urban spatial structure responds to optimal energy taxation, in both first-best and second-best cases.

More specifically, the model relies on principles from the engineering and architecture literatures by assuming that residential energy use from heating and cooling depends on a building's exposed surface area, reflecting heat transfer through exposed surfaces. According to Ching and Shapiro (2014), a building's energy use per square foot of floor space is proportional to its surface area per square foot of floor space, with surface area including the sides of the building along with the roof. Since the roof area stays constant as the height of the building increases, surface area increases less rapidly than floor space as height grows. The result is energy economies from building height, with energy use per square foot of floor space falling as height increases, a pattern seen in the empirical results of Larson et al. (2012).[3]

If a building's total energy use (and hence its total emissions) just depended on its square footage of floor space, the appropriate residential energy tax would just be a tax per square foot of space. But with surface area mattering instead, the analysis shows that residential energy taxes should include a tax per square foot of floor space *along with a tax on the building's footprint*, which captures energy usage that depends on the roof area (equal to the footprint). When buildings completely cover the land, as in the standard urban model, the footprint tax is just a tax on the entire land input, hence a land tax. By raising the land cost to the developer, this land tax encourages the construction of energy-efficient tall buildings. Note that the land tax adds to the tax burden on land already inherent in the tax on floor space.

In addition to these taxes on residential land-use, the model prescribes a commuting tax per mile to address GHG and local emissions per mile driven. This prescription emerges from a model without traffic congestion, in contrast to the work of Larson et al. (2012) and Larson and Yezer (2014), where congestion is realistically modeled.

While a carbon (or generalized emissions) tax is equivalent to a commuting tax in conjunction with the two real estate taxes, as noted above, the political feasibility of these equivalent schemes may differ, as is clear from recent experience in the United States. Imposition of a national carbon tax has been blocked, mainly by Republican opposition, but greater policy flexibility exists at the state and local levels. Moreover, that fact that real estate and gasoline taxes are already levied at the subnational level may make the use of these taxes for environmental purposes more palatable than imposition of a state or local carbon tax. Most importantly, focus on the three taxes allows exploration of partial, second-best solutions, including sole reliance on the gasoline tax in pursuit of environmental goals.

In the numerical analysis, we calibrate the model in the most realistic possible fashion and then use it to predict the impact on urban spatial structure from imposing the optimal taxes on floor space, land, and commuting. The numerical results thus allow a comparison of the optimal city, where the emissions externalities are addressed, to the laissez-faire city, where no intervention is undertaken. With emissions generated by housing consumption and commuting, the expectation is that optimal energy taxation will reduce the levels of both activities, leading to a city that is more spatially compact than a city without such taxes. The simulations show whether this broad conjecture is confirmed while illustrating the details of the city's adjustment to taxation. The results also illustrate the complementarities between the real-estate and commuting taxes, showing that, by densifying the city, the real-estate taxes reduce commuting emissions while the commuting tax similarly reduces residential emissions. These complementarities play a role in the second-best solutions, where some taxes are set at zero in the numerical analysis.

As is well known, welfare analysis in urban economics is best carried out by focusing on a fully-closed city, where resources leakages are absent (see Pines and Sadka (1986)). The rental income from land in such a city accrues to the residents rather than leaking to absentee owners, and tax revenue is also redistributed to the residents in lump sum fashion, eliminating another potential leakage. The city simulated in the paper has both these features.

One of the paper's innovations is its modeling of energy economies from building height, and this feature's connection to previous work should be noted. The models of Larson et al. (2012) and Larson and Yezer (2014) include a similar feature, although in a discrete fashion. In particular, energy use per square foot is assumed to decrease discontinuously as building height passes through several discrete critical points, in contrast to the present continuous formulation. The model of Borck (2016), by contrast, includes no energy benefits from tall buildings. His exercise of imposing building-height limits therefore generates no sacrifice on this dimension, but the resulting supply restriction, by raising the price of floor space throughout the city, reduces residential emissions by shrinking individual dwelling sizes. The urban sprawl created by height limits, however, has an offsetting effect on emissions from commuting. The present paper borrows from Borck's (2016) approach while incorporating height economies.

The plan of the paper is as follows. Section 2 presents the theoretical analysis, which includes a demonstration of the equivalence between our taxes and a carbon (generalized emissions) tax. Section 3 explains the setup of the simulation model, and Section 4 presents the simulation results. Section 5 shows results from several extensions of the model, and Section 6 offers conclusions.

2. Model

2.1. The setup

The model is based on the standard model of a monocentric city, adapted to include energy use. The model depicts a prototypical city in a high-income, industrialized country, where energy consumption is dominated by buildings and transportation (see Table 1).[4] The model portrays a closed city, with a fixed population and an endogenous level of consumer utility, rather than an open city, whose population is determined by frictionless intercity migration that ensures achievement of a given economy-wide utility level. A closed city, with its endogenous utility, is the appropriate setting for welfare analysis.[5]

In the model, the surface area of buildings, previously not an issue in urban modeling, plays a prominent role, as explained above. Suppose that buildings are square, occupying a land area of ℓ and completely covering that land, as in the standard urban model.[6] Structural density (capital per unit of land) is S and floor space per unit of land is given by $h(S)$, which is an index of the height of the building. The h function is the intensive form of a constant-returns floor-space production function, and it satisfies $h' > 0$ and $h'' < 0$.[7] Buildings contain dwellings of size q (see below), and the relation between $h(S)$ and q indicates the type of structure. If $h(S)$ is much larger than q, so that total floor space is much larger than dwelling size, then the building is high-rise multifamily structure containing many dwellings. Conversely, if $h(S)$ equals q, then the building is a single-family structure containing one dwelling. While S and q will vary with distance x to the CBD, this dependence plays no role until section 2.2 below.

Although $h(S)$ is an index of building height, the actual height is given by $h(S)$ multiplied by the height of a single storey, denoted f.[8] Therefore, each of the four sides of the building has area $fh(S)\sqrt{\ell}$ (height $\times$ width), and the area of the roof is ℓ. Surface area is then

$$4fh(S)\sqrt{\ell} \ + \ \ell. \tag{1}$$

Letting e denote energy use per unit of surface area, the building's energy use is e times (1). Energy use per unit of land is then given by

$$\frac{(4fh(S)\sqrt{\ell} \ + \ \ell)e}{\ell} \quad = \quad \frac{4fh(S)e}{\sqrt{\ell}} \ + \ e. \tag{2}$$

The second term on the RHS is energy use per unit of land due to heat transfer through the building's roof (whose area matches the lot size), while the first term captures heat transfer through the sides. It is clear from (2) that a building occupying more land has greater energy efficiency per unit of land, which would prompt the developer to increase ℓ, an incentive that is absent in the standard urban model (where ℓ is matter of indifference). However, we view ℓ as fixed and treat $A = 4f/\sqrt{\ell}$ as a parameter calibrated to generate a realistic mix of aggregate emissions from residences and commuting, as discussed below. Energy use per unit of land is then written as

$$Ah(S)e \ + \ e. \tag{3}$$

Note that the presence of the additive e term in (3) means that energy use increases less rapidly than floor space, implying energy economies from building height. Equivalently, dividing (3) by square footage ($h(S)$) shows that energy use per square foot is $Ae + e/h(S)$, an expression that is smaller in a taller building.[9]

Each unit of residential energy use generates ψ units of "composite" emissions (GHG plus local), so that (using (3)) residential emissions per unit of land are given by $\psi(Ah(S)e \ + \ e)$.

As explained in more detail below, GHG and local emissions are aggregated in a fashion that allows them to be treated as a single composite quantity.

Let the cost per unit of residential energy be denoted z, and let p denote the rental price per square foot of housing, r denote rent per unit of land, and i denote the (rental) price per unit of capital.[10] Then, energy cost per unit of land is $z(Ah(S)e + e)$ using (3), so that the developer's profit per unit of land is

$$ph(S) \; - \; iS \; - \; r \; - \; \textit{energy cost per unit of land} \;\; = \;\; (p - Aze)h(S) \; - \; iS \; - \; ze \; - \; r. \quad (4)$$

Because Aze is subtracted from p, the developer, not the tenant, pays the energy cost. In the ultimate equilibrium, however, this cost becomes embedded in p, which is endogenous.

In the absence of taxes, the developer would choose S to satisfy

$$(p - Aze)h'(S) \;\; = \;\; i, \quad (5)$$

and land rent r would be determined by the zero-profit condition:

$$r \;\; = \;\; (p - Aze)h(S) \; - \; iS \; - \; ze. \quad (6)$$

The form of both conditions is familiar from the standard urban model (see Brueckner (1987)).

Initially, e is treated as exogenous, although the model is extended below to make e another choice variable of the developer, who can reduce energy use by improving the insulation of his building and taking other costly steps. Another point to note is that e will depend on local climate. In addition, the model abstracts from the "urban heat island" effect, under which a concentration of tall buildings itself raises the ambient temperature, affecting e.[11] Similarly, the model abstracts from the light-restricting shade effects of tall buildings, which would make consumer utility inversely dependent on S (this effect has been generally ignored in urban models). Finally, it should be noted the model assumes perfect malleability of housing capital, so that the city can be rebuilt immediately in response to policy changes. Realistically, such adjustments would be expected to occur over many decades.

Energy is also used as workers commute to the CBD. Let the cost per mile of commuting (on a round-trip basis) be denoted t, so that commuting from a residence x miles from the CBD costs tx per period. The parameter t includes private energy costs, as reflected in the cost of fuel. Suppose that composite (GHG plus local) emissions per round-trip mile of commuting are given by γ, so that the energy used in commuting from a distance x generates γx worth of emissions.

Several other sources of residential energy use have been omitted from the model: kitchen appliances, such as refridgerators and stoves, and hot-water heaters. These sources can be viewed as generating a fixed amount of energy use that does not increase proportionally with the physical size of the dwelling.[12] This fixed usage presumably accounts for empirical findings showing that residential energy use per square foot of floor space falls as dwelling size rises (see, for example, Larson, Liu and Yezer (2012)). Since a city's total energy use from household appliances will thus be roughly proportional to the number of dwellings but unaffected by urban form, we omit it from the analysis. Energy used in producing the nonhousing good consumed by households (the commodity c introduced below) is also omitted from the model (Larson and Yezer (2014) include it).

2.2. Emissions and energy taxes

Let μ denote the monetary social damage from each unit of composite emissions, which is endogenous in the model, as seen below. Then, the taxes needed to support the social optimum can be derived from the model. These taxes are as follows:

- A tax of $\tau_q = \mu\psi Ae$ per square foot of floor space, addressing emissions from energy use due to heat transfer through the sides of a building
- A tax of $\tau_\ell = \mu\psi e$ per unit of land, addressing emissions from energy use due to heat transfer through a building's roof
- A tax of $\tau_t = \mu\gamma$ per mile of commuting, addressing emissions due to energy use in commuting

To demonstrate the need for these taxes analytically, let utility be given by $v(c, q, G)$, where c is consumption of a nonhousing good, q is consumption of housing floor space, and G gives the level of composite (GHG plus local) emissions affecting the city's residents. Note that, while consumers are directly affected by local emissions, they are assumed to care also about GHG emissions, reflecting an understanding of the predicted long-run climate effect of such emissions. Observe also that G is assumed to be spatially invariant within the city. While the climate effects of GHG do not vary over small areas, local pollution does vary spatially to some extent within cities, often being higher in denser areas. Since taking such variation into account would introduce substantial complexity, local pollution is instead assumed to be spatially uniform, with some loss of realism.[13] Although our modeling of local pollution thus lacks spatial detail, ignoring this type of pollution and focusing just on spatially invariant GHG emissions would not be appropriate given local pollution's importance.

Two equivalent approaches to the social planner's problem used in deriving the optimal taxes are possible, following the past literature. Under the first approach, the planner minimizes the city's resource consumption, subject to several constraints: achievement of a fixed utility level u for its residents, the requirement that the city fits its population, and a condition giving the city's endogenous level of overall emissions G.[14] Under the second approach, which is dual to the first, the planner maximizes the common utility level of urban residents subject to a resource constraint, the population constraint, and the G condition. Since the first approach is somewhat simpler, the present analysis follows it.

To start, observe that the fixed-utility constraint, which can be written $v(c, q, G) = u$ for some constant u, implies $c = c(q, G)$, with the derivatives of this function equaling minus the marginal rates of substitution: $c_q = -v_q/v_c < 0$ and $c_G = -v_G/v_c > 0$ given $v_G < 0$ (subscripts denote partial derivatives). Using the $c(q, G)$ function and letting $\overline{x}$ denote the distance to the city's edge and r_a denote the opportunity cost of land (agricultural rent), the city's consumption of resources (measured in units of c) over its circular land area is given by

$$\int_0^{\overline{x}} 2\pi x \left[iS \ + \ \frac{h(S)}{q}(c(q,G) + tx) \ + \ Ah(S)ze \ + \ ze \ + \ r_a \right] dx. \qquad (7)$$

In (7), the choice variables S and q are allowed to vary with x, while G is independent of x, as are the parameters i, t, A, z, e, and r_a. The first term within the brackets in the integrand captures capital usage, the second term equals c consumption plus commuting cost per person at distance x multiplied by the population at x. That population equals the area $2\pi x dx$ of the ring at x times h/q, where h/q gives population density (housing square footage per unit of land divided by square feet per dwelling). The remaining terms capture building energy costs and the opportunity cost of land. Table 2 provides variable definitions while containing calibration information discussed in section 3.

Letting L denote the city's fixed population, the population constraint is written

$$\int_0^{\overline{x}} 2\pi x \frac{h(S)}{q} dx \quad = \quad L, \tag{8}$$

and the multiplier associated with this constraint is λ. Total emissions G are given by

$$\int_0^{\overline{x}} 2\pi x \left(\psi[Ah(S)e + e] \;+\; \gamma \frac{h(S)}{q} x \right) dx \quad = \quad G, \tag{9}$$

where $2\pi x \gamma (x h(S)/q) dx$ gives total commute miles for consumers living in the ring at x times emissions per mile. This constraint relates the endogenous level of G to the city's characteristics, and its associated Lagrange multiplier $\mu > 0$ gives the social damage from an extra unit of emissions.

It is important to recognize that the planning solution based on (7)–(9) is implemented in each of the economy's identical cities, thus leading to nationally uniform policies that are carried out locally. An emissions policy that was instead implemented in just one city in a system of identical cities would create an incentive for intercity migration, which would then happen if cities were open. While such migration is prevented by the closed-city assumption, the incentive does not arise in the first place when identical policies are implemented everywhere, yielding the same utility change in each city.

The planner chooses values of G and $\overline{x}$ and values of S and q at each distance to minimize (7) subject to (8) and (9). After forming a Lagrangean expression using (7)–(9), the optimality conditions for S and q are generated by differentiating inside the integrals, while the condition for $\overline{x}$ comes from differentiating with respect to the limits of integration. After a modest amount of manipulation (see the Appendix), these conditions reduce to equations that identify the taxes required to support the optimum. The first equation, based on the first-order condition for q, is

$$c(q,G) \;+\; q\frac{v_q}{v_c} \;+\; (t+\mu\gamma)x \;\;=\;\; -\lambda. \tag{10}$$

Recognizing that the consumer will set v_q/v_c equal to the price p per square foot of housing, the first two terms correspond to total consumption expenditure in a decentralized equilibrium. The tx term is the money cost of commuting, but (10) shows that this cost must be supplemented by a tax of $\mu\gamma \equiv \tau_t$ per mile traveled, as in the third bullet point above. The term $-\lambda$ is constant over x and corresponds to the common income of consumers in a decentralized equilibrium. The tax τ_t is on miles driven, but since we ignore differences in automobile fuel efficiency, τ_t can be viewed as a gasoline tax.

The next condition, which is based on the first-order condition for S, is

$$\left(\frac{v_q}{v_c} - Aze - \mu\psi Ae\right) h'(S) \;\;=\;\; i. \tag{11}$$

Comparing to the profit-maximization condition (5) and recognizing $v_q/v_c = p$, (11) implies that the net price received by the developer per unit of floor space should be reduced below $p - Aze$ by the amount $\mu\psi Ae \equiv \tau_q$, a tax per square foot of floor space (as in the first bullet point above).

The laissez-faire equilibrium condition determining the distance $\overline{x}$ to the edge of the city would set r evaluated at $\overline{x}$ equal to r_a, and using (6), this condition is written

$$(\overline{p} - Aze)h(\overline{S}) \;-\; i\overline{S} \;-\; ze \;\;=\;\; r_a, \tag{12}$$

where $\overline{p}$ and $\overline{S}$ are the p and S values at $\overline{x}$. By constrast, the optimality condition for $\overline{x}$ reduces to

$$\left(\frac{\overline{v_q}}{\overline{v_c}} - Aze - \mu\psi Ae\right) h(\overline{S}) \;-\; i\overline{S} \;-\; ze \;-\; \mu\psi e \;\;=\;\; r_a, \tag{13}$$

where v_q/v_c is also evaluated at $\overline{x}$. Comparing (12) and (13) indicates that, in addition to the tax of $\tau_q = \mu\psi Ae$ per square foot of housing floor space, a tax per unit of land equal to

$\tau_\ell = \mu\psi e$ is also needed, which reduces land rent by that amount (as in the second bullet point above). With these two taxes subtracted in the equilibrium condition, it then corresponds to the optimality condition.

Note that the housing tax corresponds to a standard property tax (levied, however, as an excise tax instead of an ad-valorem tax), while the land tax matches taxes of this type levied in some cities (in excise not ad-valorem form, however). Observe also that the property tax, if levied in ad-valorem fashion, is equivalent to separate ad-valorem taxes levied at a common rate on land and housing capital (see Brueckner and Kim (2003)). An additional ad-valorem land tax would add to the tax burden, with the combined taxes equivalent to a split-rate tax structure that taxes land and capital at different rates (see Oates and Schwab (1997)). While the excise form of the current housing and land taxes disrupts this parallel, it remains true that the land tax adds to the tax burden on land already present in the housing tax.

Recall that the multiplier μ appearing in the tax terms equals the marginal social damage from emissions. From the first-order condition for G, the multiplier equals

$$\mu \quad = \quad -\int_0^{\overline{x}} 2\pi x \frac{h(S)}{q} \frac{v_G}{v_c} dx \quad > \quad 0. \tag{14}$$

The integral weights the MRS between G and c by population and sums across distance to yield the social damage from an extra unit of G.[15]

It is important to note that, because the planning problem portrays a city where the cost of land is the agricultural opportunity cost and where taxes are absent from the objective function, the corresponding decentralized city must have several features. First, the rental income generated in the city must accrue to its residents. In particular, the city must be "fully closed" in the sense of Pines and Sadka (1986), with differential land rent (the amount in excess of r_a) earned as income by the residents. The residents are thus viewed as owning a corporation that acquires the city's entire land area from its outside owners at a rental price r_a, with the land then rented to the residents themselves in a competitive market. The residents thus share the aggregate rental income net of r_a generated by the city, in effect paying rent to themselves. Second, since tax revenue is absent from the planning problem, the revenue from the energy taxes must be redistributed to the residents on an equal per capita basis. With these two requirements, the differential rent and tax revenue generated in the city stays within it, as envisioned in the planning problem. The ensuing numerical analysis imposes both requirements.

It should be stressed that the optimality of the three taxes derived in the preceding analysis holds only in the world described by the model. With a different depiction of the margins along which emissions can be reduced, additional (or perhaps different) taxes would be needed. For example, in the endogenous-e case investigated below, a subsidy on insulation costs (which reduce e) is needed or, alternatively, the optimal e can be imposed via a mandate. Or, in a model with a vehicle-type choice, the planner may wish to provide tax incentives favoring fuel-efficient vehicles.

2.3. Equivalence to a carbon (or generalized emissions) tax

The real estate and commuting taxes that have just been derived embody the tax liabilities that would be generated by a carbon (or generalized emissions) tax, making the two approaches equivalent in the context of the model. Note that since our taxes address composite emissions, a combination of GHG and local emissions, the term carbon tax is not strictly correct. To see this equivalence, note that under a generalized emissions tax, the fuels producing energy for various uses would be taxed according to the composite emissions they yield. The tax on gasoline would equal the social damage from emissions μ times the number of kilograms of composite emissions generated per gallon, denoted ξ_g. The resulting tax payment per mile of commuting would equal this expression times gallons/mile, denoted ω_g, yielding $\mu\xi_g\omega_g$. Since γ, emissions per mile, equals $\xi_g\omega_g$, this tax payment equals our commuting tax per mile, $\mu\gamma$.

Similarly, letting ξ_{hc} denote composite emissions per unit of fuel used in residential heating and cooling, the tax per unit of fuel is $\mu\xi_{hc}$. The tax per kwh of residential energy use is then $\mu\xi_{hc}\omega_{hc}$, where ω_{hc} is units of fuel per kwh. Letting $\psi = \xi_{hc}\omega_{hc}$, the tax per kwh is then $\mu\psi$. Multiplying by kwh per unit of land $(Ah(S)e+e)$, the total tax liability per unit of land is then $\mu\psi[Ah(S)e + e]$, which includes a tax of $\mu\psi e$ per unit of land and a floor-space tax liability of $\mu\psi Ae$ times floor space $h(S)$ per unit of land. Given the equivalence to a generalized emissions tax, it is appropriate to view our commuting and real estate taxes as energy taxes, even though they are levied differently.

2.4. Allowing e to be a choice variable

Suppose that instead of being exogenous, energy use per unit of surface area (e) is a choice variable of both the developer and the planner. Let $K(e)$ denote the cost per unit of surface area of achieving energy use of e (call it "insulation cost"). This function satisfies $K' < 0$ and $K'' > 0$, so that reductions in e are increasingly costly to obtain. Then, the term $z(Ah(S)e+e)$ from (7) is augmented by an additional cost term $Ah(S)K(e)+K(e)$, so that the relevant terms in the integrand of the planner's Lagrangean expression equal $(Ah(S)+1)(e(z+\mu\psi)+K(e))$. The optimality condition for choice of e is then

$$(Ah(S)+1)(z+\mu\psi+K'(e)) \quad = \quad 0, \tag{15}$$

or $K'(e) = -(z+\mu\psi)$. Since the profit-maximizing choice of e for the developer satisfies $K'(e) = -z$, his chosen e is smaller than the optimal level given $\mu\psi > 0$ and $K'' < 0$. This shortfall is a result of failure to consider environmental benefits. The optimum can be decentralized through subsidization of the insulation cost $K(e)$ at the rate $z/(z+\mu\psi)$, so that the effective cost is $zK(e)/(z+\mu\psi) < K(e)$. Alternatively, the planner could impose a mandate setting e at the level satisfying the optimality condition. Note that endogenizing e leaves the expressions for the commuting and real-estate taxes unchanged, but with e now chosen optimally. Simulation results that include this modification will be presented following the main results.

2.5. Allowing green space in the model

To allow for the possibility of urban green space, suppose that instead of covering the developed plot, the building footprint is smaller than the plot. Let the footprint be $m\ell$, with m exogenous and less than 1, and let ℓ now denote the entire plot area, which is composed of the footprint and the remaining space. This modification leaves the optimal tax per unit of footprint land unchanged, which means that the tax per unit of overall plot area falls to $m\mu\psi e$.[16]

If the uncovered plot area contains green space, residential emissions from the developed part of the plot are partly offset through absorbtion of CO_2 by the surrounding greenery. If each unit of green space absorbs σ worth of emissions, then each unit of urban land absorbs $(1-m)\sigma$ worth of emissions, where $1-m$ is the green-space share. As a result of this beneficial effect, the tax per unit of overall plot area is reduced by the amount $(1-m)\mu\sigma$, being equal to $\tau_\ell = m\mu\psi e - (1-m)\mu\sigma$. As will be seen below in the discussion of the calibrated model, this tax reduction due to CO_2 absorbtion by green space is negligible in magnitude. As a result, omitting urban green space from the model has little effect on the conclusions of the analysis.

2.6. Predicting the impact of energy taxation

A main goal of the numerical analysis presented in section 3 is to illustrate the impact on the spatial structure of the city from levying optimal energy taxes. In principle, these effects might be predictable in advance through an appropriate comparative-static analysis, relying on Pines and Sadka's (1986) extension of Wheaton's (1974) comparative-static analysis to the present context of a fully closed city.

Unfortunately, however, the required comparative statics cannot be inferred from Pines and Sadka's results. Imposition of the land tax, for example, can be viewed as equivalent to an increase in the agricultural rent r_a, which Pines and Sadka (1986) analyze. However, the present tax change corresponds to an increase in r_a combined with an increase in income equal to the rebated per capita land-tax revenue, whose impact cannot be inferred from the results they present. A similar point applies to the effects of the commuting tax. Moreover, as mentioned above, the tax on housing square footage is similar to a standard property tax, whose effects are analyzed by Brueckner and Kim (2003). While they show that the property tax causes the city to shrink spatially when the elasticity of substitution between housing and c does not exceed unity (as under the Cobb-Douglas preferences imposed below), Brueckner and Kim's model is not fully closed, nor does it incorporate redistribution of tax revenues.

The previous literature thus cannot be used directly to predict the separate effects of the three taxes in the current model, and the need to predict their *combined effects* makes the prediction task even more daunting. Hence, in the next sections, we generate results from numerical simulations.

3. Simulation Setup

3.1. Preliminaries

To evaluate the effect of imposing the optimal energy taxes, we numerically compare the urban equilibrium without any taxes to the equilibrium where the taxes are imposed, relying on the first-best optimal tax formulas. The approach thus shifts from the orientation of the planner, whose goal is to minimize the city's resource consumption, to analysis of equilibria, knowing that an equilibrium where taxes are imposed according to the optimal formulas is efficient.

The choice of an equilibrium without any taxes as a benchmark for the analysis requires some discussion. This approach matches the theory, which asks what types of taxes are needed to address emissions externalities in an otherwise undistorted economy. If pre-existing distortionary taxes were in place, then a different exercise, which would generate second-best emissions taxes, would be required. The main such distortionary tax, of course, is the existing property tax, which imposes a higher ad valorem burden than the optimal residential excise taxes in the current model (as seen below). While this tax is absent from our no-tax benchmark, we do investigate (following presentation of the main simulation results) the emissions impact of imposing a tax regime that includes a typical ad valorem property tax and a typical gasoline tax, with revenues rebated to consumers. The results provide an interesting contrast to the first-best case.

We impose specific functional forms in order to simulate the model numerically. Parameters are taken partly from published sources and partly calibrated to replicate key features of American cities. The utility function is assumed to take the following form:

$$v(c, q, G) \;\equiv\; Jc^{1-\alpha}q^{\alpha} \;-\; \nu G, \tag{16}$$

where $0 < \alpha < 1$, $\nu > 0$ is the marginal damage from composite emissions, and $J = 10^6/[(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}]$. While the first part of (16) corresponds to standard Cobb-Douglas preferences over c and q, total composite emissions G appear in the third linear term.

Using data from US metropolitan areas (MSAs), Davis and Ortalo-Magné (2011) show that the expenditure share of housing is remarkably constant across MSAs and over time, which supports the Cobb-Douglas assumption. However, the unitary implied price elasticity does not match the inelastic form of housing demand recently estimated by Albouy, Ehrlich and Liu (2016). Nevertheless, we follow many previous studies that use Cobb-Douglas preferences (e.g., Bertaud and Brueckner (2005)), setting α equal to Davis and Ortalo-Magné's (2011) estimated average expenditure share of 0.24 (see Table 2 for calibrated values). The parameter ν is set to generate a realistic value for μ, the marginal social damage from emissions (see below).

In the consumer budget constraint, income (denoted y) is set equal to the 2011 US value of median household income, equal to \$51,324. Commuting costs per mile are made up of monetary and time costs of commuting and are set at $t = \$503.53$ per mile per year (see the Appendix for details). City population is set at $L = 1.5$ million households.[17]

As in Bertaud and Brueckner (2005), housing production is assumed to be Cobb-Douglas, which yields the intensive production function $h(S) = \rho S^{\beta}$, where $\beta < 1$. Ahlfeldt and McMillen (2014) use data from several cities to estimate the elasticity of substitution between land and capital and find that it is close to one, which supports the Cobb-Douglas assumption. In the simulation, we calibrate ρ and β to generate realistic values for the average dwelling size and average commuting distance in the city, as described below. The resulting parameter values are $\beta = 0.51$ and $\rho = 0.000035$. Agricultural land rent r_a is set at \$58,800 per square mile (see the Appendix).[18]

The computation of e is unfamiliar and closely tied to the model, and it proceeds as follows. For a square k-storey building with floor space of Q, surface area is $V_k = 4kf\sqrt{Q/k} + Q/k$, where f is again the height of one storey (note that Q/k is the plot size ℓ). The Residential Energy Consumption Survey (RECS)[19] provides Q values for detached single-family homes of 1, 2, and 3 stories, and assuming $f = 12$ feet, the previous formula can be used to compute $V_k,\ k = 1, 2, 3$. Using the micro data for individual houses from the RECS, the surface-area value for each house can be computed and the median among them derived. This median surface-area value is $V = 8,458.82$. Next, we use the RECS data to get median energy use for space heating and air conditioning across all detached single-family houses (with different numbers of storeys), which equals 42,721 thousand BTUs or 12,520.29 kwh per year. We associate this median value with the median single-family surface area V, which allows us to divide 12,520.29 kwh by $V = 8,458,82$ to get a value for energy use per square foot of surface area. This value equals $e = 1.4016$ kwh/sq ft per year, and it can then be applied to buildings of all heights.[20] The price z of residential energy is set equal to \$0.062/kwh per year, using the RECS data.

As mentioned above, the value of A in (3) is set to generate realistic shares of emissions from residences and commuting. Combining the 27.1% transportation share of national emissions from Table 1 with EPA data showing that 61% of these emissions come from light vehicles,[21] the national emissions share from commuting can be approximated by $0.61 \times 27.1\%$ or 16.5%, a value that is almost the same as the residential share of 16.9% from Table 1. While we could choose A to make the city's residential and commuting emissions roughly equal, this target would ignore the commercial sector, which is not explicitly captured in the model but is important in practice. With combined residential and commercial emissions in Table 1 equal to double the residential level, we instead choose A to generate commuting emissions equal to roughly half of residential emissions, with our residential sector representing itself plus the commercial sector. The resulting value is $A = 2.75$.[22]

As noted earlier, the value of ν in the utility function is chosen to generate a realistic value for μ and thus realistic tax rates. For GHG emissions, a consensus value of μ is \$40 per metric ton of CO_2, or \$0.04/kg.[23] By appropriate choice of the units of local emissions, this \$0.04/kg applies to those emissions as well. The GHG components of γ and ψ, the commuting and residential emissions parameters, can be derived from available data, as seen in the Appendix. By applying the \$0.04/kg μ value to these γ and ψ components, the components of the tax rates τ_t, τ_q and τ_ℓ that pertain to GHG emissions follow immediately.

To derive the local emissions components of γ and ψ, we take published tax rates that are designed to correct for local emissions from commuting and residences (available from sources described in the Appendix). Each tax rate implicitly combines a social damage parameter and a local emissions value (the local component of either γ or ψ). However, by choice of units for local emissions, the social damage can be set at the same \$0.04/kg value used for GHG emissions. Knowing this μ value and the required tax rates, the local emissions components of γ and ψ can then be inferred (the units of these emissions are thus chosen). Knowing both their GHG and local components, the overall γ or ψ values are then determined, and the commuting and residential taxes follow by applying $\mu = \$0.04$/kg (see the Appendix for details).

This procedure yields $\gamma = 554.375$ kg CO_2 equivalent/mile, and multiplying by $\mu = \$0.04$/kg yields an annual commuting tax of \$22.18/mile..The procedure also yields $\psi = 0.4283$ kg CO_2 equivalent/kwh, which yields annual taxes of $\tau_\ell = \mu\psi e = \$0.04/\text{kg} \times 0.4283$ kg/kwh $\times$ 1.4016 kwh/sq ft = \$0.024/sq ft and $\tau_q = A\tau_\ell = 2.75 \times \$0.024 = \$0.066$/sq ft. Finally, the value of ν, the utility function parameter, that generates a μ of \$0.04/kg is $\nu = 0.000285$.

While the commuting tax represents 4.4% (22.175/503.53) of commuting costs t, a comparison to the existing gasoline tax gives a better sense of its magnitude.[24] The τ_t rate of \$22.175/mile per year has been scaled up by the 625 annualizing factor, and dividing by this value gives a tax of \$0.035 for each mile driven. Multiplying by 20 miles/gallon, an estimate of average fuel economy for light vehicles, this value implies a tax of \$0.71 per gallon.[25] By comparison, the average gasoline tax paid in the US is \$0.487/gallon, so that the optimal tax is about 46% larger.[26] Our tax is smaller than Parry and Small's (2005) optimal US gasoline tax, which equals \$1.01/ gallon (lying below European taxes, whose maximum rates are around \$4.00 gallon).[27] Parry and Small's larger tax, however, addresses other externalities (congestion, accidents) in addition to emissions.

Although the exposition of the model assumes for simplicity that all land is used for housing, we assume in the simulations that a fraction 0.75 of each annulus is available for residential use, with the rest devoted to roads and other public uses. In line with the suppression of the existing ad valorem property tax (as discussed above), the existing gasoline tax is eliminated by subtracting the \$0.487/gallon gasoline tax from the monetary component of the commuting-cost parameter t, leading to a net-of-tax value.

Even though our our assumed emissions damage of \$40 per metric ton (\$0.04/kg) seems to be representative of current views, some much larger values can be found in the literature.[28] Moore and Diaz (2015), for example, derive a value of \$220/metric ton, but we explore the effect of a less-extreme \$100 alternate value, or \$0.10/kg, in addition to the baseline \$0.04/kg value (close to the \$105 "high-impact" value from the Interagency Working Group on Social Cost of Carbon, United States Government (2015)).

3.2. Solution procedure

In the standard urban model (Brueckner (1987)), consumer maximization determines the housing price p as a function of utility u and the other variables of the model, given by $p(x, y, t, u)$ (to review variable definitions, see Table 2).[29] The function $q(x, y, t, u)$ gives the associated solution for housing consumption, and the developer's profit maximization problem yields analogous functions $S(x, y, t, u)$, $r(x, y, t, u)$, and $D(x, y, t, u)$, where $D = h(S)/q$ is population density.

The arguments of these functions are modified in the current framework. The utility argument is replaced by $u + \nu G$ (see (16)), and commuting cost per mile t is replaced by $t + \tau_t$ to capture the tax on commuting. In addition, letting R denote total differential land rent and T denote total tax revenue, the income y is replaced by $y + (R + T)/L$ to capture redistribution of equal per capita shares of total differential land rent and taxes.[30] Therefore, p is now written as $p(x,\ y + (R+T)/L,\ t + \tau_t,\ u + \nu G)$. In addition, the S, r and D functions now depend on this same new list of arguments along with e and τ_q.[31]

To solve the model, the first step is to set land rent at $\overline{x}$ equal to agricultural rent r_a plus the land tax τ_ℓ, with the condition written as

$$r(\overline{x},\ y + (R+T)/L,\ t + \tau_t,\ u + \nu G,\ e,\ \tau_q) \quad = \quad r_a + \tau_\ell. \tag{17}$$

This condition is used to solve for utility u as a function of the remaining variables (which include $\overline{x}$ and r_a). The u solution is then substituted back into the r, S, and D functions. When this substitution is made, G drops out as a determinant of r, S and D, but all three variables now depend on $\overline{x}$ and $r_a + \tau_\ell$, as captured in the new functions $\widehat{r}$, $\widehat{S}$ and $\widehat{D}$.

Using $\widehat{D}$, the condition analogous to (8) stating that the city fits its population is written

$$\int_0^{\overline{x}} 2\pi x\ \widehat{D}(x,\ y + (R+T)/L,\ t + \tau_t,\ e,\ \tau_q,\ \overline{x},\ r_a + \tau_\ell)dx \quad = \quad L. \tag{18}$$

An additional condition states that differential land rent integrates to R and another condition sets T equal to total tax revenue.[32] A condition defining μ, the social damage from emissions, comes from (14), and G is given by a modified version of (9).[33] The resulting set of five equilibrium conditions determines solutions for the five endogenous variables R, T, $\overline{x}$, μ, and G, with the μ solution then yielding the optimal taxes. Note that while μ is endogenous, the targeted value is \$0.04/kg, which is generated by appropriate choice of ν.[34]

To solve for the equilibrium, we use an iterative procedure. It starts with guesses for initial values of R, T and μ. Given these values, the population condition (18) is solved for $\overline{x}$. With the solution in hand, the integrals in the R, T and μ conditions are computed, using the initial guesses of R, T and μ in evaluating the integrands. The integrals then give updated values of the variables R, T and μ, which are substituted in (18), yielding a new solution for $\overline{x}$. The process continues until convergence is achieved, which occurs after relatively few iterations. The equilibrium value of G is then computed from the modified (9).

4. Simulation Results

4.1. No-tax equilibrium

We first solve for the no-tax equilibrium.[35] The procedure is to set $\tau_t = \tau_q = \tau_\ell = 0$ and then to solve (18) and the equation for R (see footnote 32) for $\overline{x}$ and R. For the no-tax city, Figures 1–5 show the spatial contours of building height $h(S)$, population density D, land rent r, the housing price p, and dwelling size q, represented by the light-blue/gray curves. Each figure shows the given variable as a function of distance to the CBD. In the figures, $\overline{x}^0$ indicates the $\overline{x}$ value in the no-tax city. In addition, Table 2 gives the central (CBD) values of these variables.

The solution gives $\overline{x}^0 = 25.43$ miles, which implies an average commuting distance of 13.74 miles, matching the average commute for workers in MSAs of 1–3 million inhabitants from the National Household Travel Survey.[36] Population (dwelling) density falls from 4224.24 dwellings per square mile at the CBD to 546.7 at $\overline{x}^0$ (average density is 738.3), and the building height index $h(S)$ falls from 0.21 square feet of housing per square foot of land at the CBD to 0.06 at $\overline{x}^0$.[37] Land rent r falls from \$24.2 million per square mile per year to $\$58,880 = r_a$. Average dwelling size is a realistic 2,196 square feet, with q rising from 1,389.14 square feet at the CBD to 3227.57 at $\overline{x}^0$. The housing price p falls from \$9.47 per square foot per year at the CBD to \$3.12 at $\overline{x}^0$. Despite the presence of rent redistribution and the emissions externality, these spatial patterns are familiar from the standard urban model.

Per-capita emissions (G/L) equal 24,164.60 kg. Residential energy use is responsible for 68% of total emissions, with commuting responsible for the balance of 32% (slightly less than half the residential amount, as targeted).

4.2. The first-best equilibrium

We now turn to the model solution when emissions taxes are levied. As explained above, the optimal taxes are given by $\tau_q^* = \$0.066$/sq ft, $\tau_\ell^* = \$0.024$/sq ft and $\tau_t = \$22.18$/mile. On average, the housing tax corresponds to an ad valorem tax of 1.17% on housing rent, the land tax amounts to an ad valorem tax of 12.93% on land rent, and the commuting tax to 4.4% of commuting costs, as mentioned above. Note that the average rates of the housing and land taxes are given by τ_q/p and τ_l/r averaged across the city's x values, while the rate of the commuting tax, which is just τ_t/t, is spatially invariant.

Table 2 gives the central values of $h(S)$, D, r, p and q in the taxed city, and Figures 1–5 show the spatial contours of these variables, which are represented by the dark-red/black curves ($\overline{x}_f$ denotes the first-best $\overline{x}$ value). The figures show that, relative to the no-tax city, the D, $h(S)$, and p contours shift up, while the q contour shifts down. The r contour rotates clockwise fashion.

In response to the optimal taxes, the city shrinks spatially, with distance to the urban boundary falling to $\overline{x}^f = 23.40$ from $\overline{x}^0 = 25.43$ miles. Compared to the no-tax case, the radius of the city thus shrinks by 8.0%, with its overall land area ($\pi\overline{x}^2$) falling by 15.4%. This finding confirms the expectation that energy taxation makes cities more compact by discouraging long commutes, reducing housing consumption, and increasing building heights. In the taxed city, population density at the CBD is 4680.74 dwellings per square mile, 11.3% higher than in the no-tax city, and density falls to 679.16 at $\overline{x}^f$ (average density is 871.54). The building height index $h(S)$ at the CBD is 0.22, 4.8% higher than in the no-tax city, and $h(S)$ falls to 0.07 at $\overline{x}^f$. Land rent r at the CBD is \$26.7 million per square mile per year, 10.3% higher than in the no-tax city, and r again falls to $r_a = 58,880$ at $\overline{x}^f$. Dwelling size q at the CBD is 1326.97 square feet, 4.5% smaller than the value in the no-tax city, and q rises to 2930.78 at $\overline{x}^f$. Average dwelling size is 2038.78 square feet, 2.2% lower than in the no-tax equilibrium. The housing price p at the CBD is \$10.05 per square foot per year, 6.1% higher than in the no-tax city, and p falls to 3.54 at $\overline{x}^f$.

Per-capita emissions G in the taxed city are 21,422.0 kg, 11.4% less than in the no-tax city. Residential energy use is responsible for 67% of total emissions, with commuting responsible for the balance of 33%.

The land-rent contour rotation in Figure 3 is similar to the one that occurs following an increase in the commuting-cost parameter t in the closed-city version of the standard model. While this t change would also lead to sympathetic rotation of the $h(S)$ and p contours in the standard case, here these contours shift up, as does the D contour. Note that higher densities offset the shrinkage of $\overline{x}$, allowing the city to still fit its fixed population.

These differences relative to the standard model show that many forces in addition to the tax-induced increase in commuting cost per mile are at work in generating the effects. These forces include responses to the land tax τ_ℓ, which tends to raise the cost of land and thus encourages developers to economize on land in production of housing, tending to raise S and the building height index $h(S)$. But since the housing tax τ_q (which is analogous to a property tax) is a tax on output of housing floor space, it tends to depress S and $h(S)$, offsetting the effect of the land tax. The housing tax also tends to reduce the dwelling size q as consumers substitute toward nonhousing consumption. The tax's effects on $h(S)$ and q, both being negative, have an ambiguous effect on population density (h/q), as discussed in detail by Brueckner and Kim (2003). These varied tax effects are mediated by the impacts of redistribution of differential land rent and tax revenue, adding to the complex interplay of forces affecting urban form in the taxed city.[38]

While the size of the commuting tax τ_t, which represents 4.4% of before-tax commuting cost, is easy to gauge, a better sense of the magnitudes of the optimal housing and land-tax rates comes from comparing them to actual US property-tax rates, expressed as a percentage of rent rather than value. Recall that a standard ad-valorem property tax is absent from the model, with the rate set to zero. Letting κ denote the property-tax rate on value and θ denote the discount rate, the property-tax rate expressed as a percentage of rent is given by $\kappa/(\kappa+\theta)$.[39] Assuming $\theta = 0.04$ and using a representative 1.5% property-tax rate (Song and Zenou (2006)), so that $\kappa = 0.015$, this expression reduces to 0.27, indicating that the existing property tax claims about 25% of rent. This rate is 23 times larger than the housing-tax rate of 1.17%, but only about double the land-tax rate of 12.93%.

The effects of a larger target value of μ, equal to \$0.10/kg, are shown in the third column of Table 2. This μ value is generated by increasing ν, the utility function damage parameter, from 0.000285 to 0.000693. As can be seen from the table, the commuting tax τ_t is now \$55.44/mile (11.0% of before-tax commuting cost), which corresponds to a large gasoline tax of \$1.77 per gallon. The housing tax rises from \$0.066 to \$0.16 per square foot, for an average ad valorem rate 2.61%, and the land tax rises from \$0.024 to \$0.06 per square foot, for an average ad valorem rate of 27.14%. Figures 1–5 actually pertain to this case because the larger impacts illustrate the qualitative effects of the taxes more clearly than in figures for the $\mu = \$0.04$ case.

In response to these taxes, $\overline{x}$ shrinks to 21.15 miles, a 17% reduction relative to the no-tax case, with the city's overall land area falling by 31%. Emissions per capita fall by 23%. CBD building height rises by 14% relative to the no-tax case, with central population density rising by 28%. Land rent at the CBD rises by 27% and the housing price rises by 16%, while the central dwelling size falls by 11%.

The benefit from imposing the optimal taxes can be gauged by computing the equivalent variation (EV) associated with the change. It equals the increase in income needed to generate the post-tax utility level when all the endogenous variables are held at the levels prevailing in the no-tax city. For the $\mu = \$0.04$/kg value, the EV is equal to 0.09% of income or \$48.15 per household per year. For the larger μ value, the EV is 0.5% of income, or \$256.35, a value similar to the benefit of correcting another distortion (road congestion) in a monocentric city, equal to 0.7% of income (as computed by Brueckner (2007)).

4.3. Complementarity of the Taxes

The residential and commuting taxes are complementary, and the numerical results can be used to illustrate this property. In particular, the commuting tax reduces residential emissions by encouraging densification of the city, and the residential taxes, by also encouraging densification, reduce commuting emissions. These outcomes can be seen in Table 4. The first two lines of the table show $\overline{x}$ along with residential, commuting and total emissions under the no-tax and first-best equilibria. The next two lines show outcomes where either the commuting tax or the residential taxes are set at zero, with the remaining tax(es) set at first-best value(s). The third line of the table shows that when the residential taxes are set at first-best values and $\tau_t = 0$, both types of emissions drop along with $\overline{x}$. The decline in commuting emissions equals 83% of its first-best decline even though commuting is not taxed, and the drop in residential emissions is 66% of the first-best decline. Conversely, when the commuting tax is set at its first-best value and the residential taxes equal zero (line 4 of the table), 17.7% of the first-best decline in residential emissions is achieved, while the commuting emissions reduction is 27% of the first-best decline. Overall, the table shows that the residential taxes are more important than the commuting tax in reducing emissions from both sources. As will be seen, the tax complementarities seen in Table 4 play a role in the second-best analysis in the next section.

4.4. Second-best optima

Under second-best optima, one or two of the three taxes is constrained to equal zero, with the remaining tax(es) set to maximize utility.[40] Rather than attempting to characterize the second-best optimal taxes analytically, the taxes are found by a search procedure. Although we have analyzed various second-best optima, attention is restricted to the most interesting among them, in which the housing and land taxes are set at zero and the commuting tax is used by itself to address the emissions externality. Note that, although this case resembles line 4 of Table 3, the difference is that τ_t is now chosen optimally rather than set at the value from the first-best optimum. In computing the second-best optimal τ_t and the resulting equilibrium, the utility-function parameter ν is set at the original value of 0.000285, which is consistent with a μ of \$0.04/kg in the first-best case.

When both the housing and land taxes are set at zero, the optimal τ_t equals \$79.68/mile, or 15.8% of average commuting costs. This large tax increase relative to the first-best value of \$22.18/mile is needed both to replace the lost effect of the residential taxes on commuting emissions as well as to limit residential emissions themselves (complementarity effects like those mentioned above). As seen in the fourth column of Table 3, the urban boundary in this second-best case is at $\overline{x} = 24.09$ miles, 3.9% above the first-best level. Central buildings are taller (0.23 vs. 0.22), and central density is 5.5% higher, than in the first best. Central housing prices and land rents are 2.8% and 10.8% higher, respectively, than in the first best, and central dwellings are 9.8% smaller. The exaggerated responses of these central values match the impact of a higher τ_t in the standard model, where it raises central land and housing prices, building height and population density via clockwise rotations of the spatial countours for the first three of these variables. Per-capita emissions are 3% higher than in the first best. The EV associated with this second-best policy is 0.06% of income, or \$32.70 (appropriately smaller than the 0.09% first-best value).

If policy makers were to consider use of existing urban taxes to counteract a city's emissions, it would be natural for them to focus on the gasoline tax, not heeding this paper's prescription for additional taxes on land and housing. The resulting tax of \$79.58/mile translates to a gas tax of \$2.55/gallon, like the high levels seen in Europe. Therefore, as a result of opting not to levy housing and land taxes, policy makers would need to raise the gasoline tax \$1.84 beyond the first-best level of \$0.71.[41]

4.5. Impact of existing taxes

As mentioned above, the model can be used to analyze the emissions impact of the current tax structure. To do so, the commuting tax is set a value corresponding to the \$0.487/gallon tax, yielding $\tau_t = 15.22$. In addition, to capture the ad valorem property tax, the optimal excise taxes on housing and land are dropped and replaced with a 27% ad valorem tax on housing rent, which corresponds to a typical 1.5% tax on value (see above). While the developer's net revenue per square foot of housing is $p - Aze - \tau_q$ with the excise tax, it equals $(1 - \tau_q)p - Aze$ with an ad valorem tax on rent at rate τ_q. As before, revenues from both taxes are returned to consumers in lump-sum fashion.

Under this tax structure, the city shrinks spatially, with $\overline{x}$ falling to 23.62 miles (0.8% higher than in the first best), and per capita emissions decline to 21,360.1 kg, a drop equal to 102% of the first-best decline. This larger-than-optimal decline is composed of a drop in residential emissions larger than the first-best decline, reflecting the higher tax burden on housing, along with an insufficiently large drop in commuting emissions, consistent with the lower-than-optimal gas tax. Because of the distortion inherent in the ad valorem property tax, consumer utility falls under this tax structure despite the reduction in emissions, with the equivalent variation equal to -0.28% of income. See Table 3 for additional information.

4.6. Sensitivity analysis

This subsection provides sensitivity analyses. We increase population and income by 50% of the benchmark values, with the results shown in Table 5. Note that, since ν is held fixed at the original value, the given parameter changes will cause the emissions damage μ to diverge from its previous target value. The changes in the tax rates (which all occur in the same proportion) reflect the change in μ.

First, we increase income by 50% from the benchmark value of $51,324, to $76,986. This value corresponds to the household income in very rich metro areas such as San Francisco or Boston. In the standard urban model, such an increase leads to higher average housing consumption, longer commutes, and urban sprawl. Obviously, these effects increase emissions. In the first-best city, the income increase leads to a 32.2% increase in $\overline{x}$ and a 60% increase in emissions per capita relative to the benchmark first-best city. Interestingly, the optimal taxes each fall by 1.2%, reflecting the same decrease in μ.

Next, we increase population from $L = 1,500,000$ to 2,250,000. Comparing first-best cities, the population increase leads to a 8.4% increase in $\overline{x}$. Emissions per capita decrease by 11.2%, following a pattern seen in Larson and Yezer (2015), where emissions per capita fall with city size in the absence of taxes.[42] Each of the taxes increases by 54.9%, reflecting a large increase in μ. Further sensitivity analysis involves increases in t, e, and γ, with predictable results that are available on request.

5. Extensions

5.1. Making e endogenous

Up to this point, residential energy use e per square foot of surface area has been treated as exogenous. As explained in section 2.4, however, e can be a choice variable of the developer and the planner, with the developer's profit-maximizing e (which ignores environmental benefits) smaller than the planner's optimal value. Recall that $K(e)$ gives the (insulation) cost per unit of surface area of achieving energy use of e. To simulate the outcome with an endogenous e, we represent $K(e)$ by Be^{-k}, with $k > 0$.

This function is calibrated as follows. First we set the parameter k at a plausible value ($k = 0.5$),[43] and we then calibrate B by requiring that the existing e value of 1.40 kwh/sq ft satisfy the developer's first-order condition $-kBe^{-k-1} = -z$. With the abatement cost function in hand, we then compute e^*, the socially optimal e, which satisfies $-kBe^{*-k-1} = -(z + \mu\psi)$. This e value is $e^* = 1.1914$, equal to 85% of the existing e, representing an improvement in energy efficiency.

The model is simulated again, with the baseline equilibrium involving no taxes and the existing e (also, $\mu = 0.04$/kg). In the first-best comparison case, e is set at e^* by mandate, and the optimal taxes are imposed. Importantly, the optimal real estate taxes are based on e^* rather than e, being equal to $\tau_q = \mu\psi Ae^*$ and $\tau_\ell = \mu\psi e^*$.

For these simulations, we keep the previous parameterization of $h(S)$ rather than recalibrate to maintain a realistic commute distance. Because the abatement costs make development more expensive than before and $h(S)$ is the same, the no-tax and first-best cities are more compact than previously (with a shorter average commute), and they have higher prices, taller buildings and smaller dwellings.

The no-tax and first-best results are shown in the last two columns of Table 3. While the commuting tax is the same as before, the smaller e^* means that both real-estate taxes are lower than in the second column of the table. In addition, the first-best city is spatially smaller than the no-tax city, and it has taller buildings and smaller dwellings at the center along with higher central density, land rent, and housing prices. Reflecting the improvement in residential energy efficiency, the decline in emissions per capita relative to the no-tax city, from 14,173.60 to 12,235.40 kg, is slightly larger in percentage terms than before: 13.7% vs. the 11.4% value from the first and second columns of the table. The EV, at 0.12% of income, is also somewhat larger than the 0.09% value in original first-best case. Overall, the results in Table 2 show that the main conclusions of the analysis are robust to incorporating an endogenous e.

5.2. Adding urban green space

As explained in section 2.3, if the developed plot contains extra space in addition to the building footprint, then the land tax rate equals $m\mu\psi e$, where m is the footprint share of plot area. If the extra area is green space, containing CO_2-absorbing greenery, then the land-tax rate is reduced to $m\mu\psi e - (1-m)\mu\sigma$, where σ is absorbtion per square foot of green space.

To gauge the size of the tax reduction due to green space, results in Velasco, Roth, Norford and Molina (2016) can be used to estimate σ. Their findings indicate that green space in two California cities absorbs as much as 0.90 tons of CO_2 per hectare annually, or 0.0084 kg/sq ft. Using this σ value and assuming $m = 0.5$ for illustrative purposes, the implied tax per square foot of land is $(1/2)(\mu\psi e - \mu \times 0.0084) = (1/2)(0.024 - 0.0034) = (1/2)(0.0237)$. Therefore, the land tax is only slightly lower [(1/2)(0.0237) vs. (1/2)(0.024)] than if greenery was absent from the vacant plot area. The upshot is that modifying the model to include vacant plot area, which is then assumed to contain green space, would not materially change the conclusions regarding the effects of imposing optimal taxes.

5.3. Polycentric city

We have assumed a monocentric city, with all jobs located in the CBD. Clearly, this assumption is at odds with the reality of modern cities, where employment tends to be dispersed. We now briefly discuss how the model's results are affected by allowing for dispersed employment. Detailed results are available in an online Appendix.

For simplicity, we assume a linear city with the CBD at $x = 0$ and the endogenous city border still denoted by $\overline{x}$ (we consider the right side of the city only). Note first that our optimal tax formulas are exactly the same as before. To see why, suppose we exogenously put a secondary business district (SBD, which offers the CBD wage) at distance x_s from the CBD. Then, letting $x_1 \equiv x_s/2$, residents located between 0 and x_1 commute to the CBD and residents located between x_1 and x_s and between x_s and $\overline{x}$ commute to the SBD (note that CBD and SBD incomes net of commuting costs are equal at x_1). Looking at equations (7)–(9) in the text, the term tx in (7) in will be replaced by $t\widetilde{x}$ and the term $\gamma(h(S)/q)x$ in (9) will be replaced by $\gamma(h(S)/q)\widetilde{x}$, where $\widetilde{x}$ is the distance to the CBD or SBD, whichever is closer.[44] It is then easy to see that the optimal tax formulas are unchanged.

Our findings change quantitatively, however, with the online Appendix showing detailed results from a numerical simulation. The main finding is that a city with subcenter has shorter commuting distances, which reduces total emissions. Even though better job access reduces land rents and housing prices, making buildings shorter and dwellings larger and thus raising residential emissions, the net effect on total emissions is negative. We find that polycentricity leads to lower optimal tax rates, since the marginal damage of emissions is lower. The percentage reductions in emissions and the city's spatial size in moving to the first best are consequently smaller than in a monocentric city.

6. Conclusion

This paper has presented the first investigation of the effects of optimal energy taxation in an urban spatial setting where emissions are generated by both residences and commuting, using a model that incorporates emissions economies from tall buildings. The first-best optimal tax structure in the model has taxes on housing, land and commuting, which match the effects of a carbon or generalized emissions tax. Since emissions come from housing consumption and commuting, optimal taxation reduces the levels of both activities, generating a more-compact city with a lower level of emissions per capita. In response to optimal taxation, the spatial area of the city shrinks by 15.4%, with its central population density rising by 11.3%. Emissions per capita fall by 11.4%, a notable reduction. While these impacts are based on a representative value of the social damage from emissions, larger effects on urban structure emerge when the optimal taxes are based on a higher damage value.

Use of three separate taxes rather than a generalized emissions tax allows the paper to carry out a second-best exercise, which sets the housing and land taxes at zero, so that the commuting tax must do all the work in limiting emissions from both residences and commute trips. In this case, the second-best optimal commuting tax would correspond to a gasoline tax of $2.55 per gallon, more than five times the current US average tax and $1.84/gallon above the first-best optimal tax of $0.71/gallon (which is complemented by optimal housing and land taxes).

Future research could add traffic congestion to the model, following the lead of Larson et al. (2012). While they realistically assume that congestion affects emissions through lower travel speeds, a simpler approach would keep the current link between emissions and distance traveled, with congestion just raising commuting costs. Then, congestion charges, which would depend on endogenous traffic volumes, would supplement the current commuting tax. It seems likely that, by densifying the city and thus reducing both residential and commuting emissions, congestion charges would reduce the social damage μ from emissions, leading to smaller values of τ_q, τ_ℓ, and τ_t.[45] Thus, by leading to the same kinds of changes in urban form as the current taxes, congestion charges would allow a reduction in their levels.

Another extension would explore more general forms for consumer preferences, replacing the convenient Cobb-Douglas form with realistically calibrated CES preferences. Such a change, however, is unlikely to significantly alter the main lessons of the paper. In addition, following Larson and Yezer (2014), the emissions generated by nonhousing consumption could be added to the model. Finally, the use of zoning regulations in place of taxes could be explored, recognizing that the first-best outcome can be exactly achieved by location-specific regulations on dwelling size and building height (following the planner's first-order conditions for these variables). Larson et al. (2012), Larson and Yezer (2015), and Borck (2016) have explored the effect of less-draconian but more-realistic policies such as green belts, density limits and a city-wide height limits, but further investigation may be useful.

An additional issue concerns the assumption that consumers understand the predicted impacts of GHG on climate, thus having GHG in their utility function. If consumers instead cared only about local pollution (having a short time horizon, for example), the optimal tax rates would be lower and the tax effects on urban structure smaller. Generally, the question of how to treat consumer attitudes toward climate change in modeling optimal policies is a profound one that goes far beyond the current paper.

Appendix

A1. Planning-problem derivations

The Lagrangean expression for the planning problem is generated by subtracting the RHS expressions in (8) and (9) from the left-hand sides, multiplying the resulting expressions by the multipliers λ and μ, and adding (7). The first-order conditions for S, q, G and $\overline{x}$ are

$$S: \qquad i \;+\; \frac{h'(S)}{q}[c(q,G)+tx] \;+\; Ah'(S)ze \;+\; \lambda\frac{h'(S)}{q} \;+\; \mu\psi h'(S)e \;+\; \mu\gamma\frac{h'(S)}{q}x \;=\; 0 \qquad (a1)$$

$$q: \qquad -\frac{h(S)}{q^2}[c(q,G)+tx] \;-\; \frac{h(S)}{q}\frac{v_q}{v_c} \;-\; \lambda\frac{h(S)}{q^2} - \mu\gamma\frac{h(S)}{q^2}x \;=\; 0 \qquad (a2)$$

$$G: \qquad -\int_0^{\overline{x}} 2\pi x\frac{h(S)}{q}\frac{v_G}{v_c}dx \;-\; \mu \;=\; 0 \qquad (a3)$$

$$\overline{x}: \qquad i\overline{S} \;+\; \frac{h(\overline{S})}{\overline{q}}[c(\overline{q},G)+t\overline{x}] \;+\; Ah(\overline{S})ze \;+\; ze \;+\; r_a \;+\; \lambda\frac{h(\overline{S})}{\overline{q}} + \mu\psi[Ah(\overline{S})e+e] \;+\; \mu\gamma\frac{h(\overline{S})}{\overline{q}}\overline{x} \;=\; 0. \qquad (a4)$$

Rearranging ($a2$) yields (10), and ($a3$) is the same as (16). Solving ($a2$) for λ and substituting in ($a1$) yields (11) after rearrangement, and substituting in ($a4$) yields (13) after rearrangement.

A2. Data sources and calibration calculations

Income y is set at the 2011 value of median household income in the US, which is \$51,324 (the source is American Community Survey of the US Census Bureau).[46] To compute commuting cost per mile, t, we follow Bertaud and Brueckner (2005). We use the median hourly wage of \$17.09 (from Bureau of Labor Statistics)[47] and value it at 50% (Small (2012)) to get an hourly time cost of commuting \$8.545. Assuming that rush hour traffic moves at 30 miles per hour, the implied time cost of commuting is \$0.28/mile. As for the money cost of commuting, the current Federal allowance is \$0.55/mile, which includes an average gasoline tax of \$0.025/mile (\$0.487/gallon divided by the average light-vehicle fuel economy of 20 miles per gallon). Subtracting this amount yields a net-of-tax money cost of \$0.525/mile and an overall commuting cost per mile of \$0.805. Multiplying by 1.25 workers/household, by 250 work days/year and again by 2 to convert to a round-trip basis, annual commuting cost per mile is \$503.125/year.

The computation of agricultural rent r_a again follows Bertaud and Brueckner (2005). We take the average value of farm real estate per acre in 2011, \$2300 (the source is United States Department of Agriculture (2015), Land Values: 2015 Summary, `http://www.usda.gov/nass/PUBS/TODAYRPT/land0815.pdf`). To convert this number to annual rent, we use a discount rate of 4% to get a rent per acre of $\$2,300/0.04 = \$92$, yielding a land rent per square mile of $r_a = \$58,880$.

To derive GHG and local emissions from commuting, we use data from the National Research Council (2010), along with a standard estimate of GHG damage equal to \$40/metric ton CO_2, or \$0.04 per kg CO_2. NRC (2010), Table 4-5 (p. 180), gives 0.552 kg CO_2/mile as GHG emissions from gasoline, and valuing these emissions at \$0.04/kg gives GHG damage per mile of \$0.02208. If GHG damage were the only damage, this number (converted to an annual basis) would correspond to τ_t. Local damage exists as well, however, and NRC estimates this damage as \$0.0134/mile. Local damage can be viewed as the product of local commuting emissions per mile, γ_l, and social damage per unit of local automobile emissions, μ_l^{com}, which must satisfy $\mu_l^{com}\gamma_l = \$0.0134$/mile. However, by choice of units of local pollution, we can set μ_l^{com} equal to \$0.040/kg, the same damage as per unit of GHG emissions, and then use the previous equation to determine γ_l, which equals $0.0134/0.04 = 0.335$ kg/mile. Therefore, composite emissions from commuting consist of 0.552 kg CO_2/mile of GHG emissions and 0.335 kg/mile of local emissions, for a total of 0.887 kg/mile, with both valued at \$0.04/kg. Converting the 0.887 value to an annualized per mile value by multiplying by 625 ($2 \times 500 \times 1.25$) yields $\gamma = 554.375$/mile, and using $\mu = \$0.04$, the annual first-best commuting tax is $\tau_t = \$0.04 \times 554.375 = \$22.175$/mile.

Turning to residential emissions, we use the Residential Energy Consumption Survey to apportion total BTUs of household energy use for space heating and air conditioning (converted to kwh) across five sources: electricity, natural gas, propane/LPG, and fuel oil and diesel/kerosene.[48] Then, from Carbon Trust,[49] we get CO_2 generation per kwh of energy for the five sources: 0.5246 kg CO2e/kwh for electricity, 0.1836 for natural gas, 0.2147 for LPG, 0.2674 for fuel oil, 0.2517 for diesel/kerosene. Multiplying by kwh for each source and summing gives total residential CO_2 generation, and dividing by total residential kwh gives CO_2 generation per kwh of residential energy use. This quantity is 0.1997 kg CO_2/kwh, which equals the ψ value for GHG emissions. Again valuing these emissions at $\mu = \$0.04$/kg (\$40/metric ton), and multiplying by $e = 1.4016$ kwh/sq ft would give the floor space and land taxes for GHG emissions.

However, the local emissions component of composite residential emissions remains to be considered. NRC (p. 235) gives \$0.016/kwh as the local emissions damage from electricity generation, while the spreadsheet from Parry et al. (2014) gives local damage from the natural gas used in heating as \$0.322/GJ or \$0.00116/kwh.[50] We weigh these values by the adjusted electricity and natural gas proportions in heating and cooling from the RECS (ignoring the other energy sources), which equal 53.79% and 46.21% respectively. The resulting local residential emissions damage is then \$0.00914/kwh. As in the case of commuting, this damage is the product of a local ψ, denoted ψ_l, and a social damage per unit of local residential emissions, μ_l^{res}, whose product must satisfy $\mu_l^{res}\psi_l = \$0.00914$. As before, we can choose the units of local residential emissions so that the social damage μ_l^{res} per unit is the same \$0.04/kg value as for GHG emissions. The implied value of ψ_l is then given by $\psi_l = 0.00914/0.04 = 0.2285$ kg/kwh. Adding this value to the ψ value of 0.1997 for GHG emissions gives an overall ψ equal 0.2285 + 0.1997 = 0.4283 kg CO_2/kwh. The overall first-best residential taxes are then $\tau_\ell = \mu\psi e =$ \$0.04 $\times$ 0.4283 kg/kwh $\times$ 1.4016 kwh/sq ft= \$0.024/sq ft and $\tau_q = A\tau_\ell = 0.066$/sq ft.

Table 1: 2013 Emissions by Sector (millions of metric tons CO_2 equivalent)

Electricity-generation emissions are distributed to final user

implied sector	*volume*	*percentage*
Industry	1922.6	30.0%
Transportation	1810.3	27.1%
Residential	1129.1	16.9%
Commercial	1126.7	16.9%
Agriculture	646.4	9.7%
Total	6673.0	100%

Columns do not sum since emissions from US Territories are excluded. Source is Environmental Protection Agency (2015, Table ES-7)

Table 2: Variable definitions

Variable	*Definition*	*Units*	*Depends on distance x?*	*Calibrated value*
x	Distance from CBD	miles	–	–
$\overline{x}$	Distance to city's edge	miles	–	–
ℓ	Building plot area	square feet	no	–
S	Structural density (housing capital per unit of land)	tons/sq ft	yes	–
$h(S)$	Square feet of housing per unit of land (height index)	square feet/sq ft	yes	–
q	Housing consumption (dwelling size)	square feet	yes	–
c	Nonhousing consumption (annual)	dollars	yes	–
u	Consumer utility	–	no	–
p	Price per unit of housing (annual)	dollars/sq ft	yes	–
r	Rent per unit of land (annual)	dollars/sq mi	yes	–
r_a	Agricultural land rent (annual)	dollars/sq mi	no	\$58,000
i	Capital rental price (annual)	dollars/ton	no	1
y	Household income (annual)	dollars	no	\$51,354
t	Commuting cost (annual)	dollars/mi	no	\$503.53
L	City population	households	–	1.5 million
D	Population density	households/sq mi	yes	–
e	Energy use per unit surface area (annual)	kwh/sq ft	no	1.4016
z	Residential energy cost (annual)	dollars/kwh	no	\$0.062
f	Floor height	feet	no	–
A	$4f/\sqrt{\ell}$	–	no	2.75
α	Housing exponent in utility	–	no	0.24
β	Exponent in h function	–	no	0.51
γ	Auto emissions (CO_2 equiv.)	kg/mi	no	554.375
ψ	Residential emissions (CO_2 equiv.)	kg/kwh	no	0.4283

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Footnotes

[*]We thank Howard Chong for steering us toward the engineering/architecture literature on residential energy use, and we are grateful to Sofia Franco, Georg Hirte, Tatsuhito Kono, Will Larson, Pierre Picard, Kevin Roth, Ken Small, and Tony Yezer for detailed comments and to Ben Leard and other conference participants for additional comments. We also thank Don Fullerton and several referees for extremely helpful feedback. Any shortcomings in the paper, however, are our responsibility.

[1]See Borck and Pflüger (2015) for a multi-city analysis with global emissions, as well as Gaigné, Riou and Thisse (2012).

[2]In similar fashion, Fullerton and West (2002) show that, in treating automobile emissions, an emissions tax can be replaced by taxes with other features that acheive the same outcome (i.e., a gas tax that depends on fuel type, engine size, and installed pollution control equipment, or a vehicle tax that depends on mileage).

[3]Their results show that, holding dwelling size constant, energy use is 6% lower in a 2-4 unit structure than in a single-unit structure and 32% lower in structures with 5 or more units.

[4]Our theoretical model cannot capture specific characteristics of a particular U.S. city. Since cities differ widely, we find it most useful below to calibrate our model to match a hypothetical city with average US characteristics.

[5]In an open city, imposition of taxes would typically lead to out-migration, an outcome that is prevented in a closed city.

[6]Among buildings with a given footprint area, square buildings have the smallest surface area (see below).

[7]The primitive CRS production giving output of floor space is written $H(N, \ell)$, where N is capital, and dividing the arguments by ℓ allows the function to be written as $\ell h(N/\ell) \equiv \ell h(S)$, where h gives floor space per unit of land.

[8]Imagine constructing a building on 1 square foot of land ($\ell = 1$), containing 1 square foot of floor space. The height f of the walls of this (tiny) building should be enough to accommodate the usual ceiling height of 8-9 ft, leading to an f value of, say, 12 ft. Now suppose that another square foot of floor space is added, again holding the land area fixed at 1 sq ft. This addition requires a second storey (necessitating a larger increment of capital

given $h'' < 0$), with another 12 feet added to the wall height. Thus, the building height is $fh(S) = 12 \times 2$.

[9] It should be noted that this formulation may overstate energy economies from tall buildings by ignoring the energy cost of vertical travel due to electricity usage by elevators. More generally, urban models do not include time spent in vertical travel as an element of mobility costs within cities.

[10] The capital price i is normalized to 1 without loss of generality. Its value in fact has no impact on the urban equilibrium.

[11] See Borck (2016) for an application of this heat island effect.

[12] Energy use from appliances (and lighting) may, of course, show a modest increase with dwelling size (from larger refridgerators and hot-water heaters or additional televisions), but omission of this effect is acceptable as an approximation.

[13] In the model of Schindler et al. (2017), which focuses on local pollution, the level of pollution varies spatially.

[14] See Fujita (1989) for another use of this approach.

[15] The dual version of the planning problem starts by deriving income-compensated demand functions for c and q conditional on G, denoted by $c(p, G, u)$ and $q(p, G, u)$. In (7)–(9), the first function is substituted in place of $c(q, G)$ and the second is substituted in place of q. Then, (7) is set equal to I, which gives the economy's total endowment of c. Finally, u is maximized subject to the three modified constraints, with p, G, and $\overline{x}$ treated as choice variables along with u. The optimality conditions in (10), (11) and (13) again emerge. See Pines and Sadka (1986) for another use of this approach.

[16] This modification requires multiplying the $2\pi x$ term in (7) by m (recognizing that the housing-related terms are on a building-footprint basis), while replacing r_a in (7) by r_a/m, so that the outside m factor cancels, leaving r_a unmodified. Since the integrands in (8) and (9) are also multiplied by m, this factor cancels in deriving the optimality conditions involving S and q. In the optimality and equilibrium conditions for $\overline{x}$, the only change is that r_a is replaced by r_a/m, so that the tax per unit of footprint area remains the same.

[17] With an average household size of 2.6, the city would then have 3.9 million residents.

[18]For consistency of units of measurement, housing output and dwelling size in the simulation are measured in square miles, although the results, including those for p, are presented on a square foot basis (in the computations, e is also expressed on a square mile basis).

[19]The survey can be found at `http://www.eia.gov/consumption/residential/`.

[20]By ignoring the possible irregular shapes of single-family houses, this calculation may lead to a biased value of e, but the result is acceptable as an approximation.

[21]See `https://www.epa.gov/greenvehicles/fast-facts-transportation-greenhouse-gas-emissions`.

[22]Assuming $f = 12$, setting $A = 4 \times 12/\sqrt{\ell}$ equal to 2.75 yields a value of $\sqrt{\ell}$ (the dimensions of the square plot) of 17.4 feet, an unrealistically small value. However, more-complex building shapes, which would raise the 4 factor in A, could generate a more realistic plot size. In any event, the ℓ is roughly of the right order of magnitude (being off by a factor of six or seven).

[23]See Interagency Working Group on Social Cost of Carbon, United States Government (2015) (`https://www.whitehouse.gov/sites/default/files/omb/inforeg/scc-tsd-final-july-2015.pdf`).

[24]Comparison of the residential taxes to existing property taxes is carried out below.

[25]US Department of Transportation data at the following link show miles per gallon for the US fleet of cars and light trucks of 23 and 17, respectively. With light trucks constituting about 40% of the overall light vehicle fleet (White (2004)), average miles per gallon is around 20. (`http://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/publications/national_transportation_statistics/html/table_04_23.html`)

[26]See the following webpage from the American Petroleum Institute: `http://www.api.org/oil-and-natural-gas-overview/industry-economics/fuel-taxes/gasoline-tax`.

[27]See the following webpage from US Department of Energy: `http://www.afdc.energy.gov/data/10327`.

[28]Nordhaus (2014), for example, reports estimates using different assumptions that range between $21 and $104 per metric ton.

[29]Under the Cobb-Douglas preferences in (16) (with $\nu = 0$), $p(x, y, t, u) \equiv B(y - tx)^{1/\alpha} u^{-1/\alpha}$,

where B is a constant.

[30]In reality, tax revenues might be used to subsidize energy-efficient public transit or building modifications designed to reduce energy use. Analysis of these options would require a more detailed model.

[31]The dependencies of S can be seen in (11), where $\mu\psi Ae = \tau_q$ and the MRS expression is replaced by the modified p function. The S that satisfies the equation then depends on the arguments of p and on e and τ_q (D inherents these dependencies). The r dependencies can be seen from the LHS of (13). Land rent r is given by the LHS expression in (13) with p in place of the MRS and the bars removed, so that r depends on the arguments of p along with e and τ_q.

[32]The R condition is written as

$$R \;=\; \int_0^{\overline{x}} 2\pi x\; [\widehat{r}(\cdot) - r_a]dx,$$

where the arguments of $\widehat{r}$ are suppressed (note that R appears on both sides of this condition). The condition giving total tax revenue is

$$T \;=\; \tau_t \int_0^{\overline{x}} 2\pi x \widehat{D}(\cdot) x dx \;+\; \tau_q \int_0^{\overline{x}} 2\pi x h(\widehat{S}(\cdot))dx \;+\; \tau_\ell \int_0^{\overline{x}} 2\pi x dx,$$

where the arguments of $\widehat{S}$ and $\widehat{D}$ are suppressed. Note that, since T appears in these arguments, it is present on both sides of this condition.

[33]The μ condition is

$$\mu \;=\; -\int_0^{\overline{x}} 2\pi x \widehat{D}(\cdot)\widehat{MRS}(\cdot)dx,$$

where $\widehat{MRS}(\cdot)$ is the function corresponding to $v_G/v_c = -\nu/v_c$, which has the same list of arguments as $\widehat{D}$. In deriving the G condition, S in the first term of (9) is replaced by $\widehat{S}(\cdot)$ and h/q is replaced by $\widehat{D}(\cdot)$. Since G does not appear in the arguments of $\widehat{S}$ and $\widehat{D}$, the modified (9) thus gives G in terms of the other endogenous variables, whose values are determined by the previous conditions.

[34]It should be noted that endogeneity of the taxes would be eliminated if the $\widehat{MRS}$ expression in footnote 33 (and in the original equation (14)) were a constant. This case emerges, for example, if preferences over c and q in (16) take the Leontief form, making v_c a constant, denoted ϕ. With v_G equal to the constant ν, the MRS is then ν/ϕ and footnote 33 gives $\mu = L\nu/\phi$, yielding exogenous taxes via the tax formulas.

[35]It could be argued that ν should be chosen to yield the target value of μ in the no-tax equilibrium rather than in the first best, given that this equilibrium matches the current real-world one. However, under the chosen ν, the no-tax equilibrium generates a μ almost identical to the target value of \$0.04/kg, making this choice moot.

[36]See `http://nhts.ornl.gov/`.

[37]This $h(S)$ value is unfortunately unrealistic, being more appropriate for a suburban house than a central-city building (the value 0.21 corresponds to a single storey house cover one-fifth of the lot area). Efforts to adjust the model's calibration could not eliminate this discrepancy. While the model thus cannot replicate the level of building heights realistically, it still credibly shows the qualitative impact of taxation on the height measure, as seen below.

[38]In all MSAs with populations between 1.5 and 2.5 million, the average population density is 540 people, or 208 households, per square mile (`www.census.gov`). While this population range is below the population of the simulated city, the 540 density value should be representative of that in a somewhat larger city.

[39]It is interesting to ask whether non-tax interventions could guide the city toward a first-best outcome. One such intervention is suggested by Joshi and Kono's (2009) demonstration that a combination of minimum and maximum height restrictions for buildings can be used to address land-use distortions generated by traffic congestion. Following their logic and referring to Figure 1, imposing appropriate minimum height limits (which follow the dark first-best curve) in the central part of the city and imposing maximum height limits in the outer part of the city could generate the optimal building-height pattern. However, since dwelling sizes would remain uncontrolled, these limits would not generate a first-best outcome.

[40]To derive this expression, note that property value P is determined by the relationship $P = (p - \kappa P)/\theta$, with P equaling the discounted value of the flow of rent minus taxes. Solving yields $P = p/(\kappa + \theta)$, so that the tax liability as a percentage of rent is given by $[\kappa p/(\kappa + \theta)]/p = \kappa/(\kappa + \theta)$.

[41]For a similar exercise in the context of automobile pollution, see Fullerton and West (2010).

[42]In another second-best exercise, Borck (2016) studies building-height limits as a different tool for combating global warming. The intuition is that, by tightening housing supply, lower building heights may depress housing consumption, thus reducing emissions. However, the population-density contour in a city with building-height limits is too flat, compared to a city with first-best taxation.

[43]Their result emerges when the population increase is caused by an exogenous increase in amenities in an open-city context.

[44]Changing the value of k has little effect on the results.

[45]Formally, $\widetilde{x} = x$ if $x \leq x_1$ and $\widetilde{x} = |x_s - x|$ otherwise.

[46]In other words, holding the utility parameter ν fixed at the value that produces $\mu = \$0.04/kg$ in the current setup, adding congestion and congestion charges would lead to a lower μ.

[47]The source is available at `https://www.census.gov/prod/2013pubs/acsbr12-02.pdf`.

[48]See `http://www.bls.gov/oes/current/oes_nat.htm`.

[49]While the raw data are used for this computation, the average figures are shown in line 1 of Table CE4.1.

[50]See Carbon Trust, Conversion factors: Energy and carbon conversions, 2011 update (`http://www.carbontrust.com/media/18223/ctl153_conversion_factors.pdf`).

[51]The spreadsheet can be found at `http://www.imf.org/external/np/fad/environ/data/data.xlsx`.