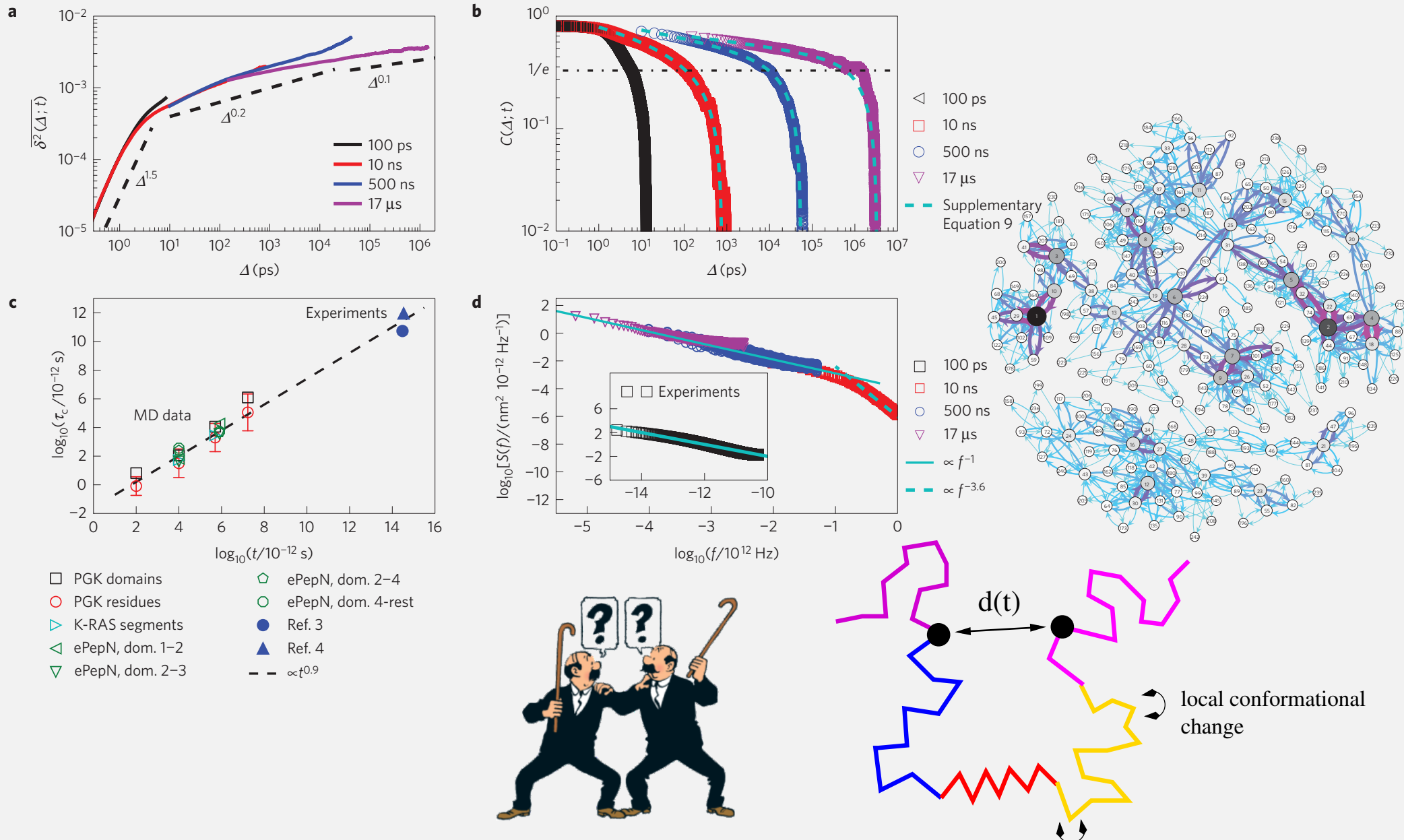


Anomalous diffusion & ageing in membranes

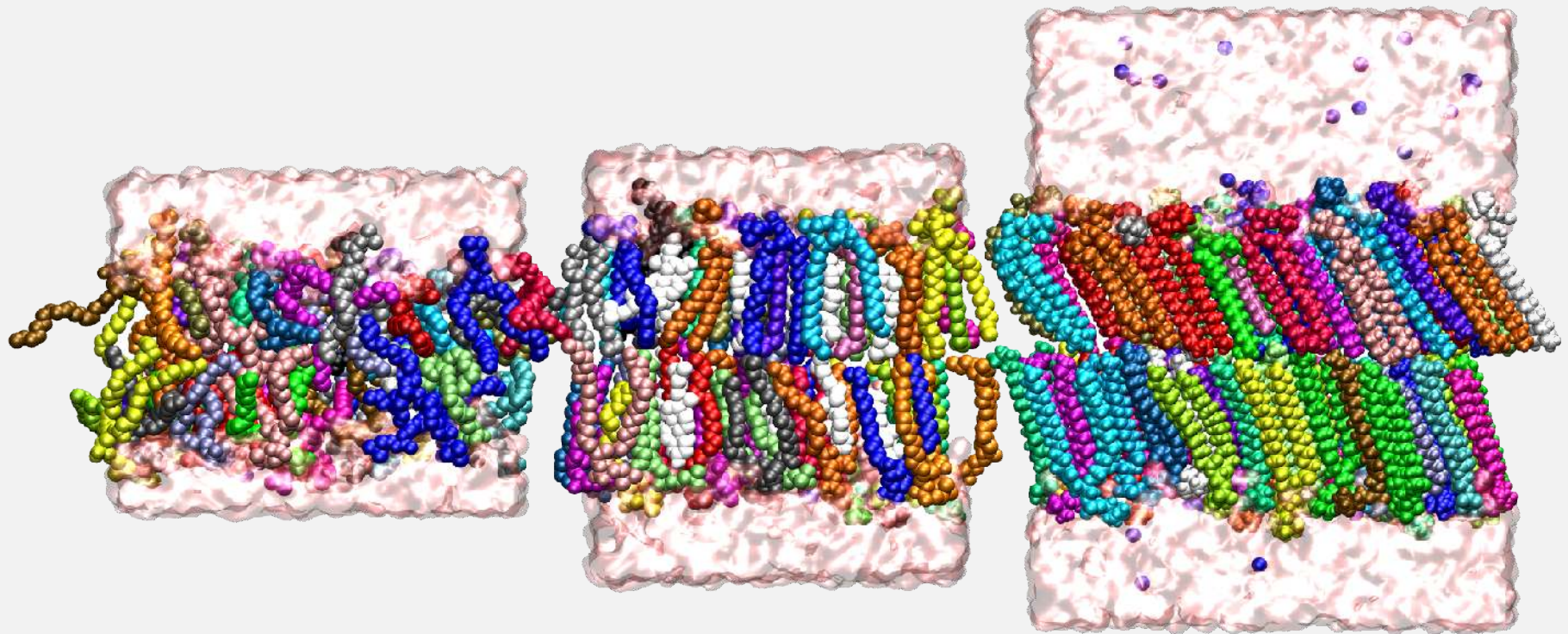
— POSTECH, 30th November 2016 —



Self-similar internal dynamics: ageing over 13 decades



Single lipid motion in bilayer membrane MD simulations

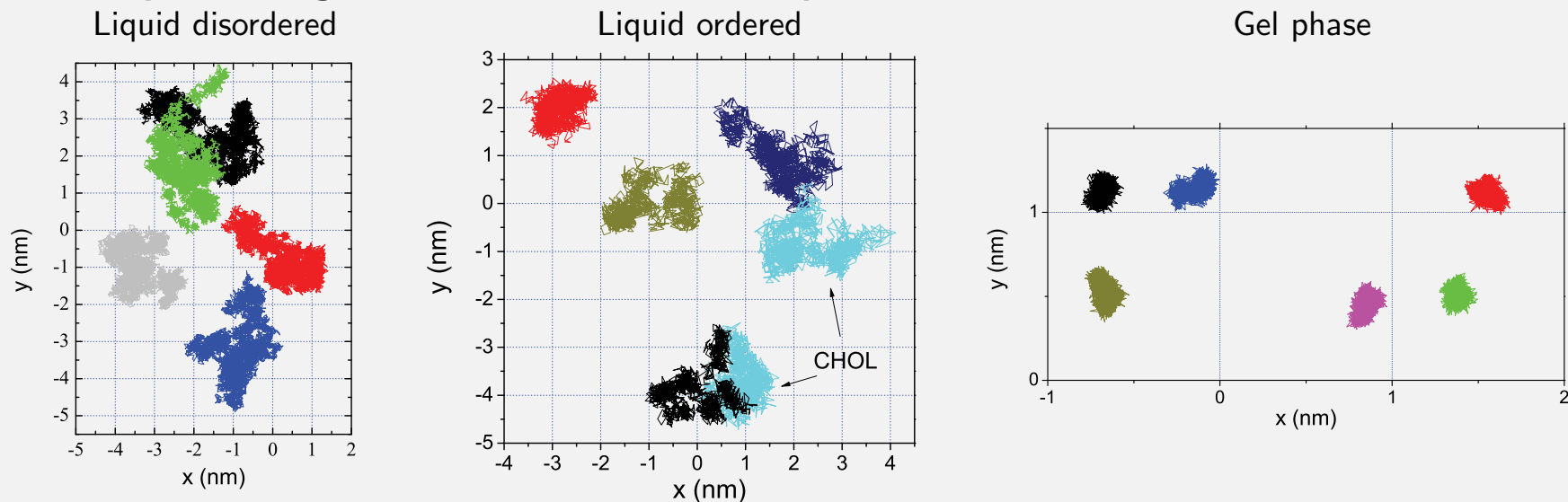


Liquid disordered

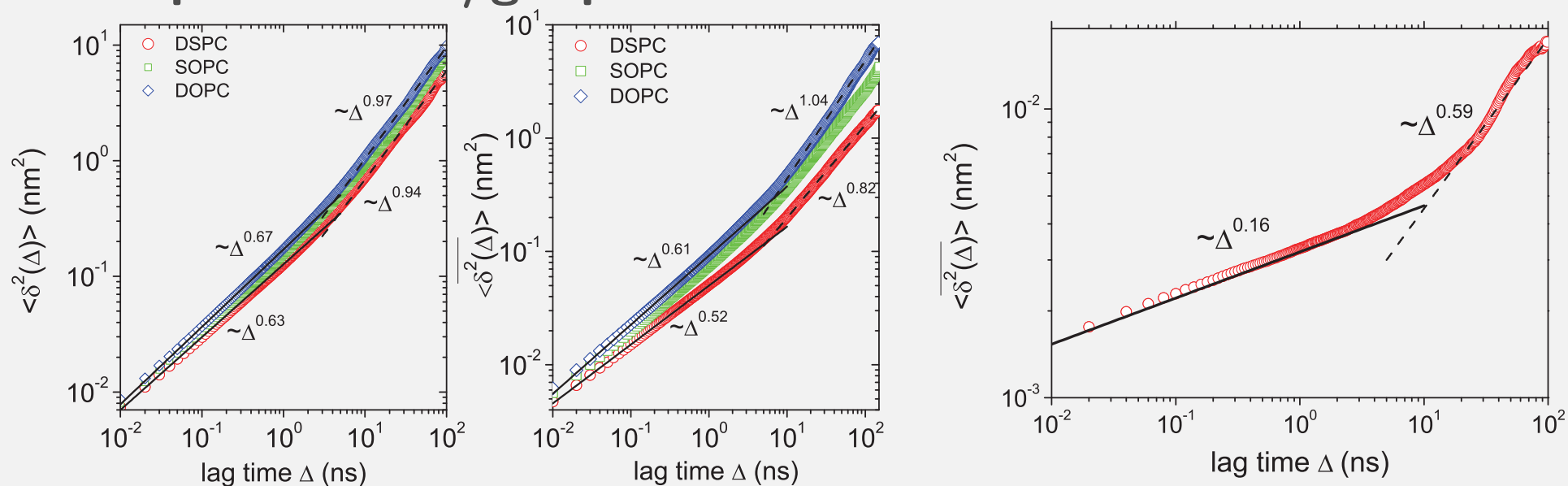
Liquid ordered

Gel phase

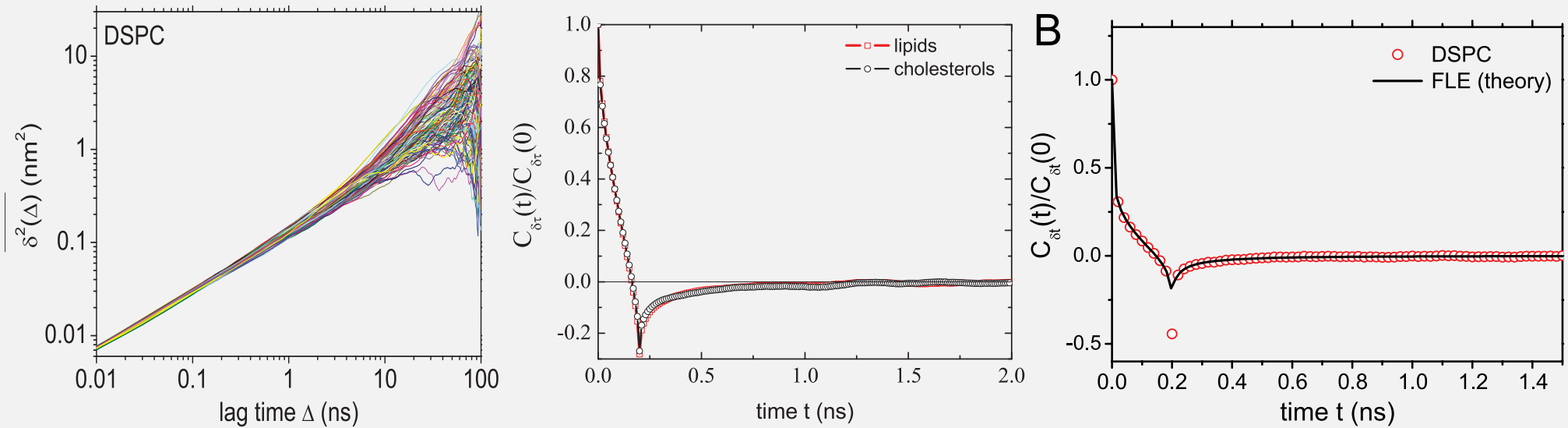
Sample trajectories for the lipid & cholesterol motion



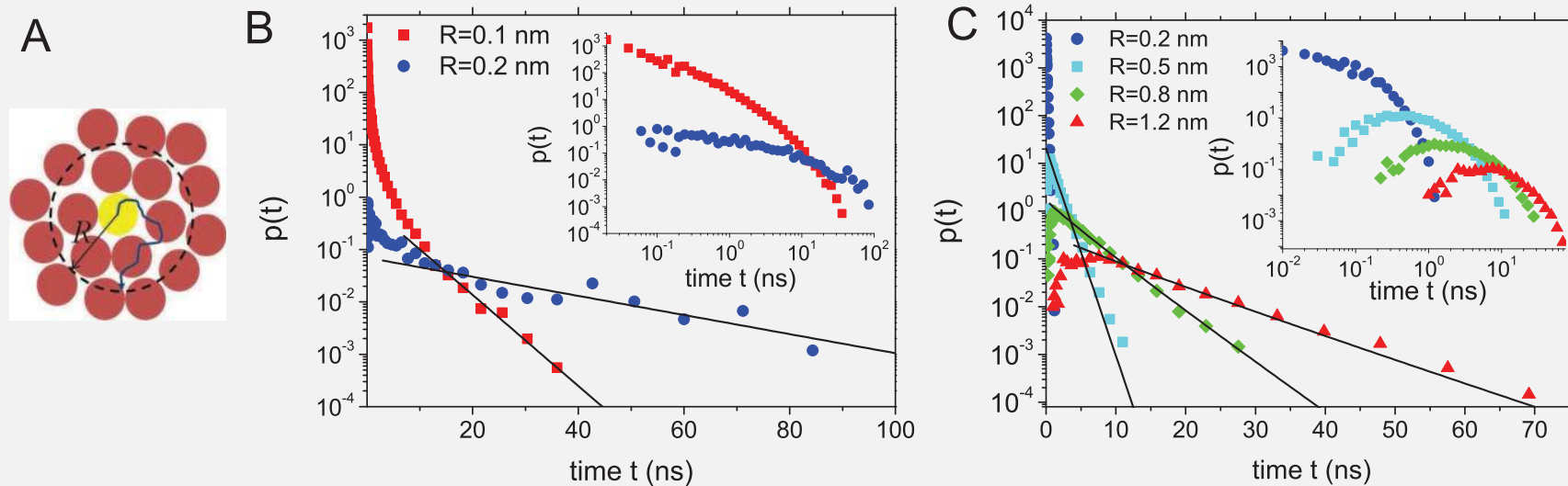
Liquid ordered/gel phases: extended anomalous diffusion



Reproducible TA MSD & antipersistent correlations



Rattling dynamics: exptl first passage PDF $\rightarrow$ FLE motion



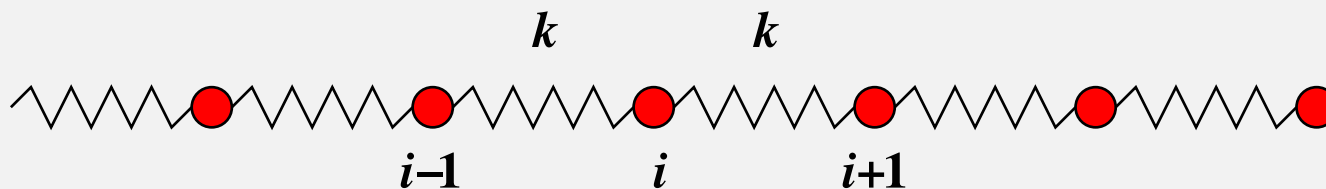
Generalised Langevin equations in viscoelastic systems

Coupled set of Markovian processes (e.g., Rouse model for polymers):

$$m_i \ddot{\mathbf{r}}_i(t) = k(\mathbf{r}_i - \mathbf{r}_{i+1}) + k(\mathbf{r}_{i-1} - \mathbf{r}_i) - \eta \dot{\mathbf{r}}_i + \sqrt{2\eta k_B T} \times \boldsymbol{\zeta}_i(t)$$

Integrating out all d.o.f. but one $\leadsto$ Generalised Langevin equation (GLE):

$$m \ddot{\mathbf{r}}(t) + \int_0^t \eta(t-t') \dot{\mathbf{r}}(t') dt' = \mathbf{f}(\mathbf{r}, t) + \boldsymbol{\zeta}(t) \therefore \eta(t) = \sum_{i=1}^N a_i(k) e^{-\nu_i t} \rightarrow t^{-\alpha}$$



Kubo fluctuation dissipation theorem (in conti limit $\eta(t) \simeq t^{-\alpha}$ fractional Gaussian noise):

$$\langle \zeta_i(t) \zeta_j(t') \rangle = \delta_{ij} k_B T \eta(|t - t'|)$$

overdamped limit: FBM

Quantum mechanics: Nakajima-Zwanzig equation using projection operators

Hydrodynamics: Basset force with $\eta(t) \simeq t^{-1/2}$ due to hydrodynamic backflow

Membranes can be supercrowded

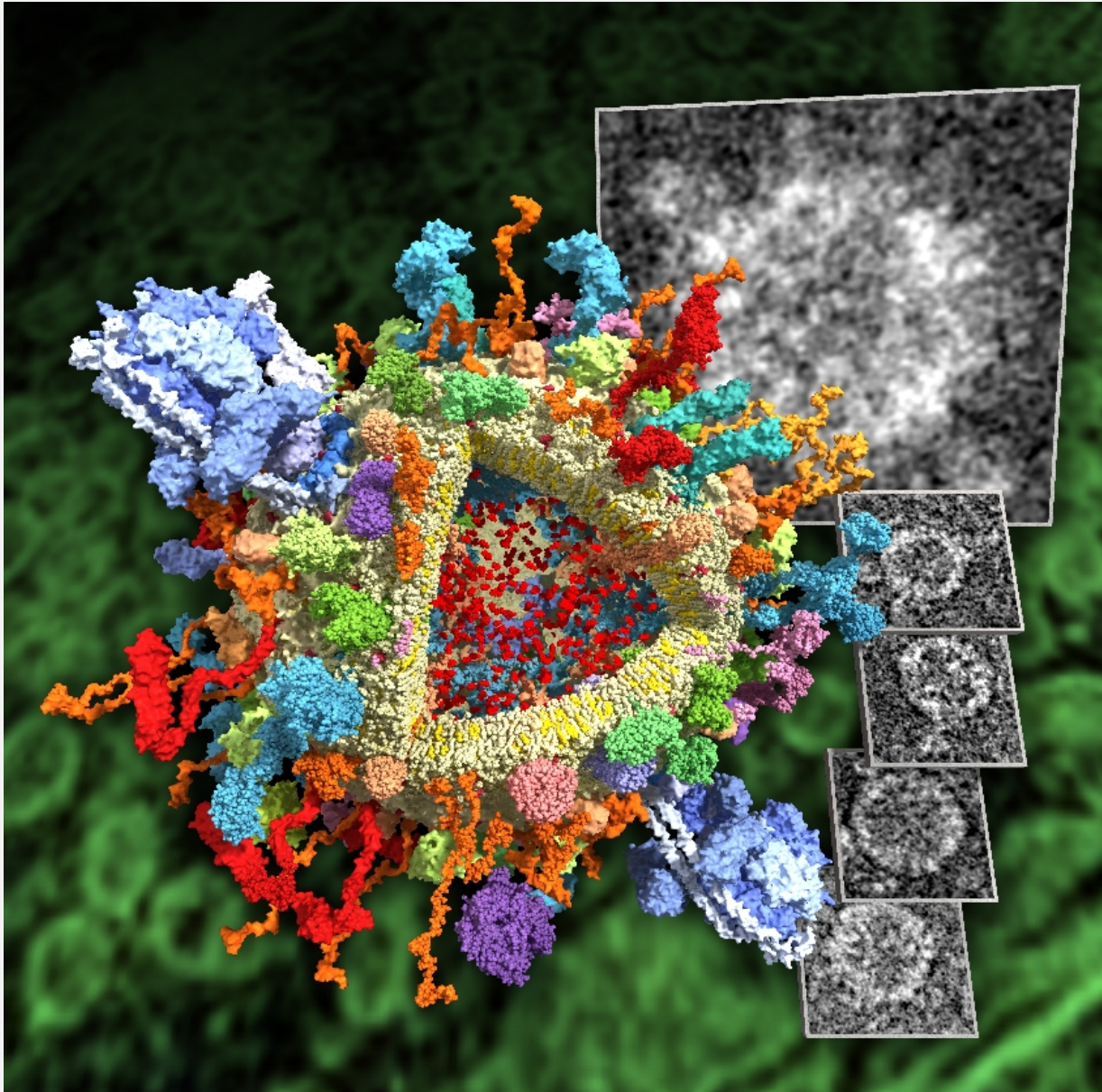
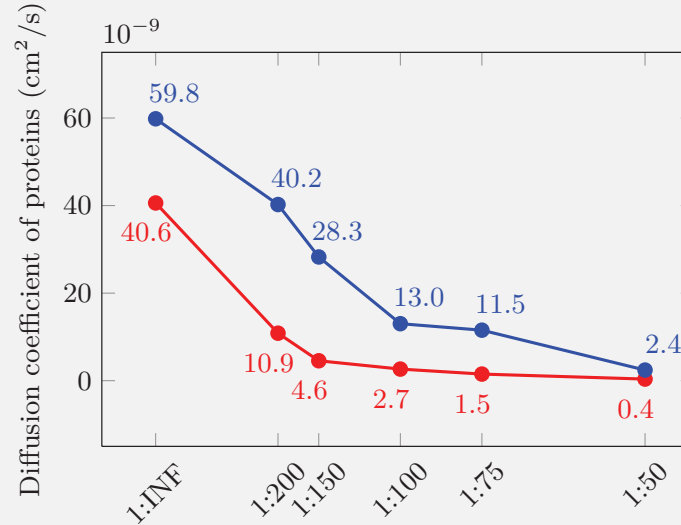
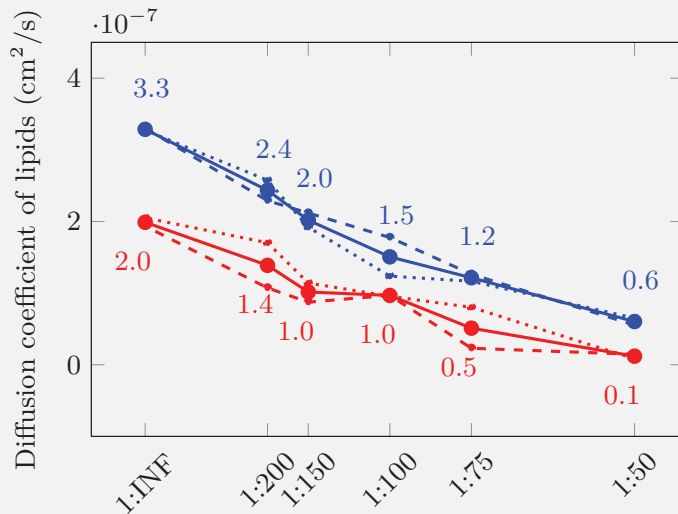
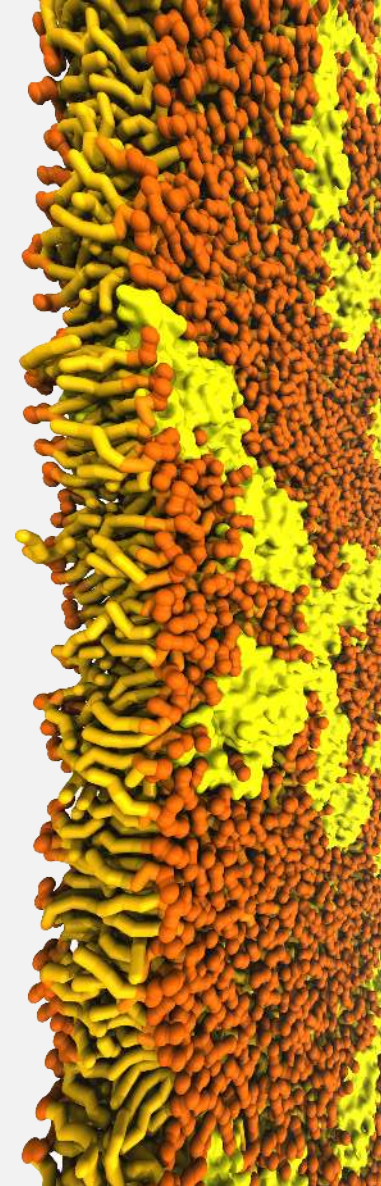


Figure courtesy Helmut Grubmüller

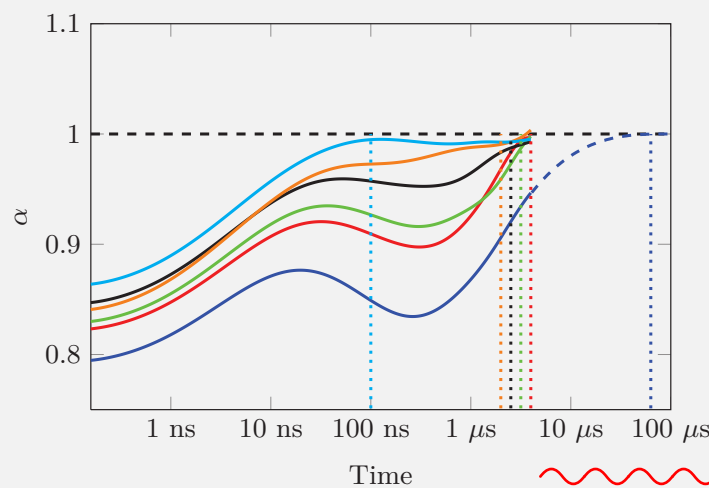
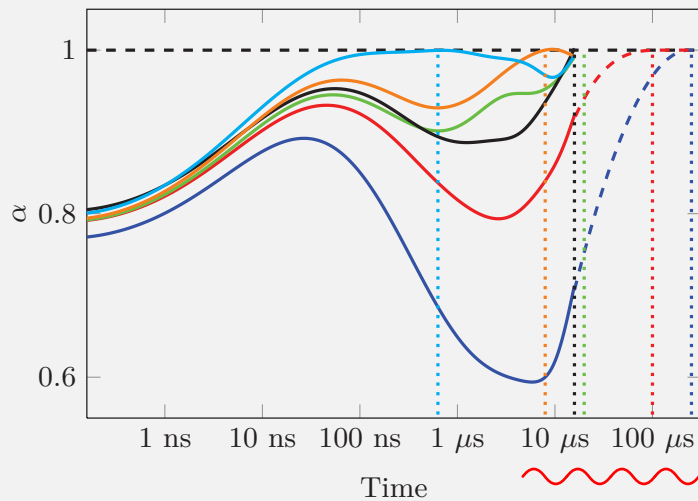
Protein crowded membranes reduce effective mobility



Blue: DLPC. Red: DPPC

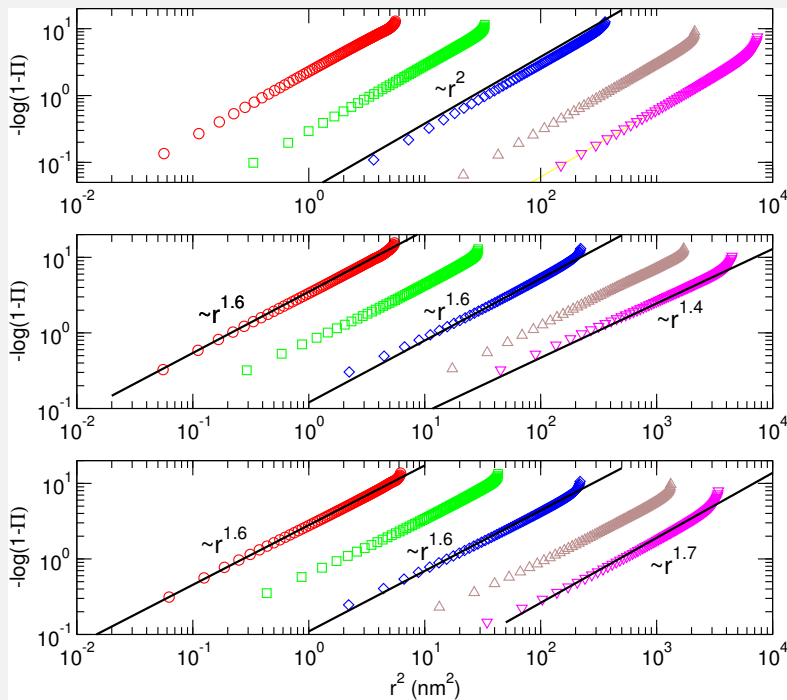
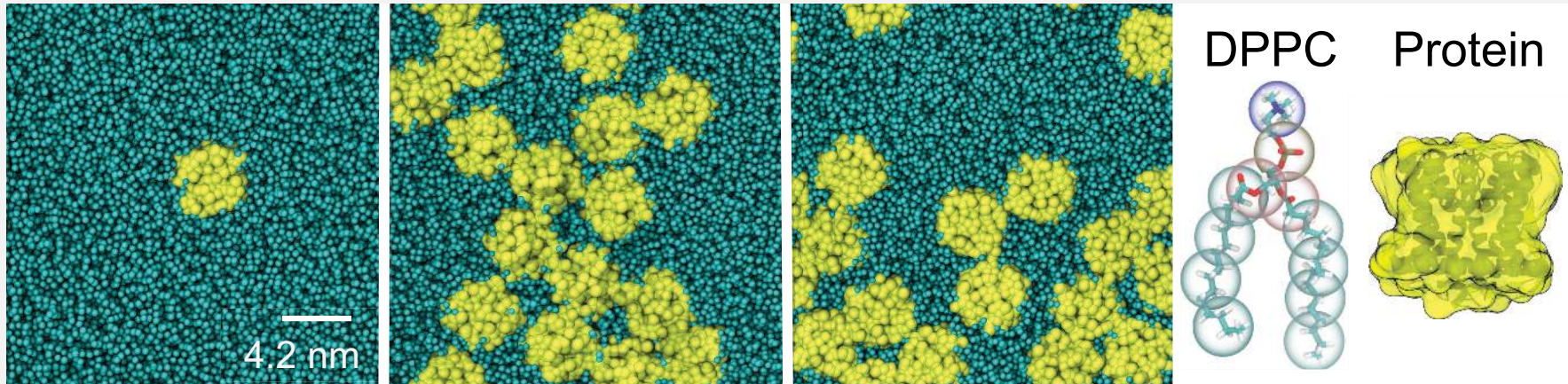


Protein crowding effects anomalous lipid diffusion



Left: DPPC (protein-aggregating) case. Right: DLPC protein non-aggregating case.

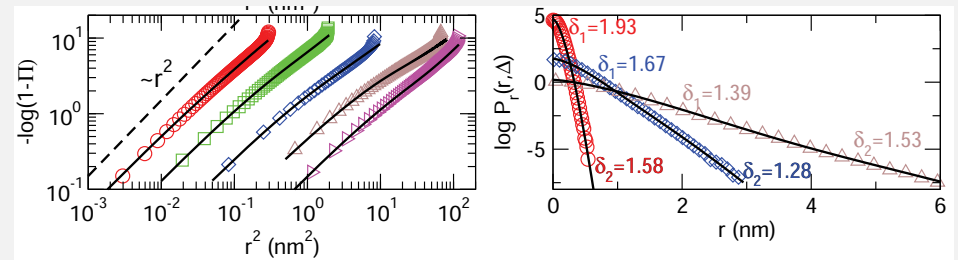
Crowding in membranes: non-Gaussian lipid/protein diffusion



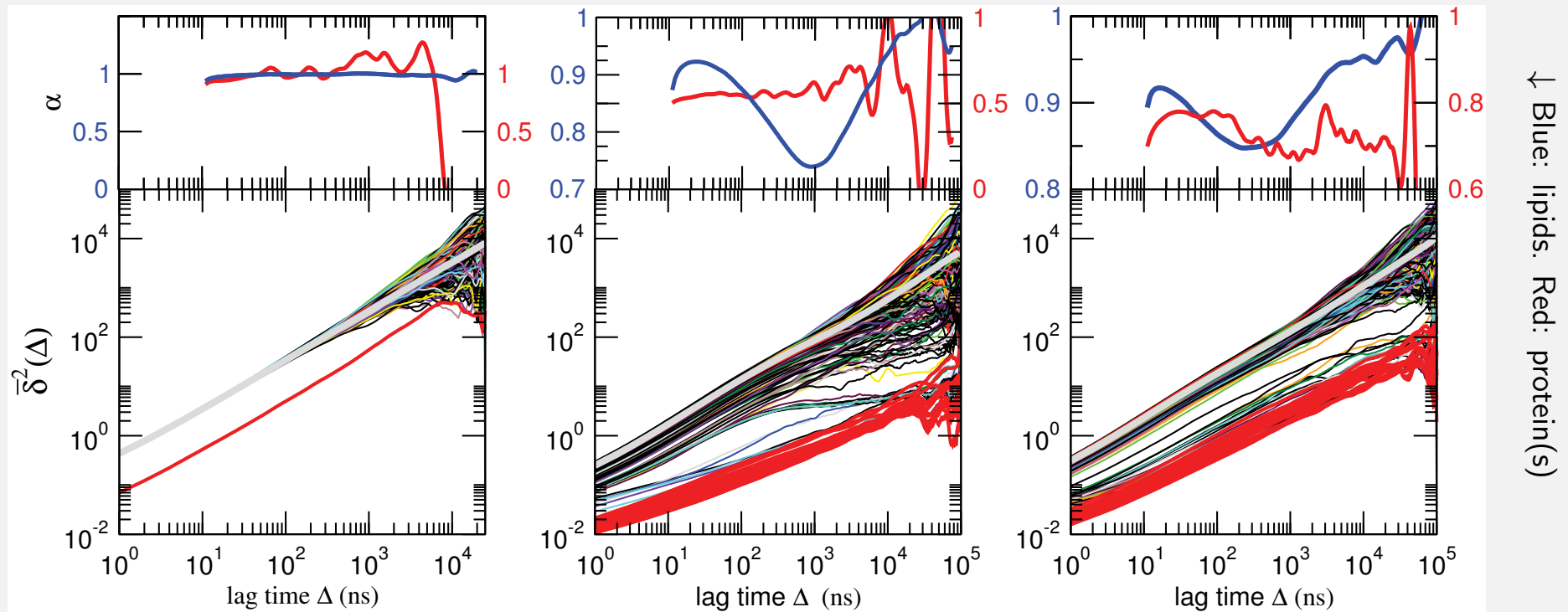
Dilute membrane: $P(r, t)$ Gauss

Crowded membrane ($\delta \approx 1.3 \dots 1.7$):

$$P(r, t) \propto \exp \left(- \left[\frac{r}{ct^{\alpha/2}} \right]^\delta \right)$$



Crowding in membranes increases dynamic heterogeneity



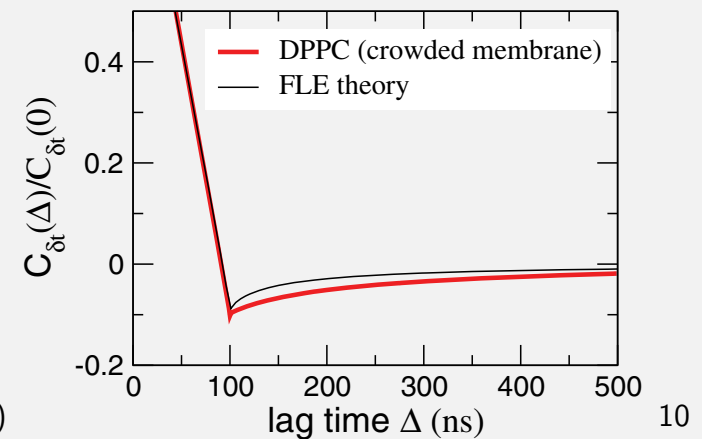
Single NaK channel

DLPC (non-aggregating)

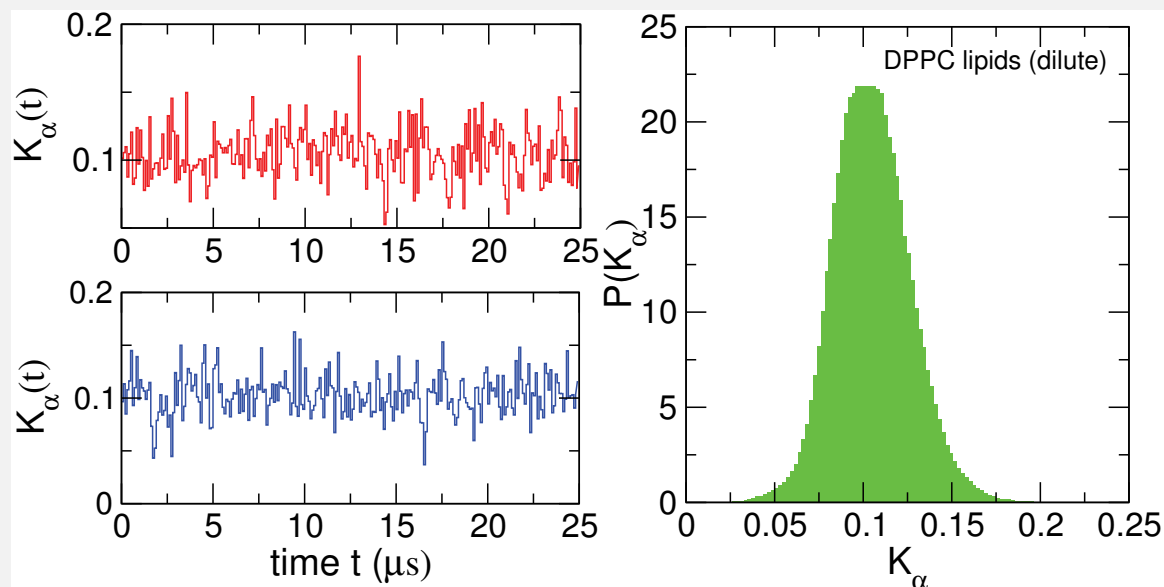
DPPC (aggregating)

Lipids & proteins behave quite differently

Increment correlation no longer simple FBM $\rightarrow$

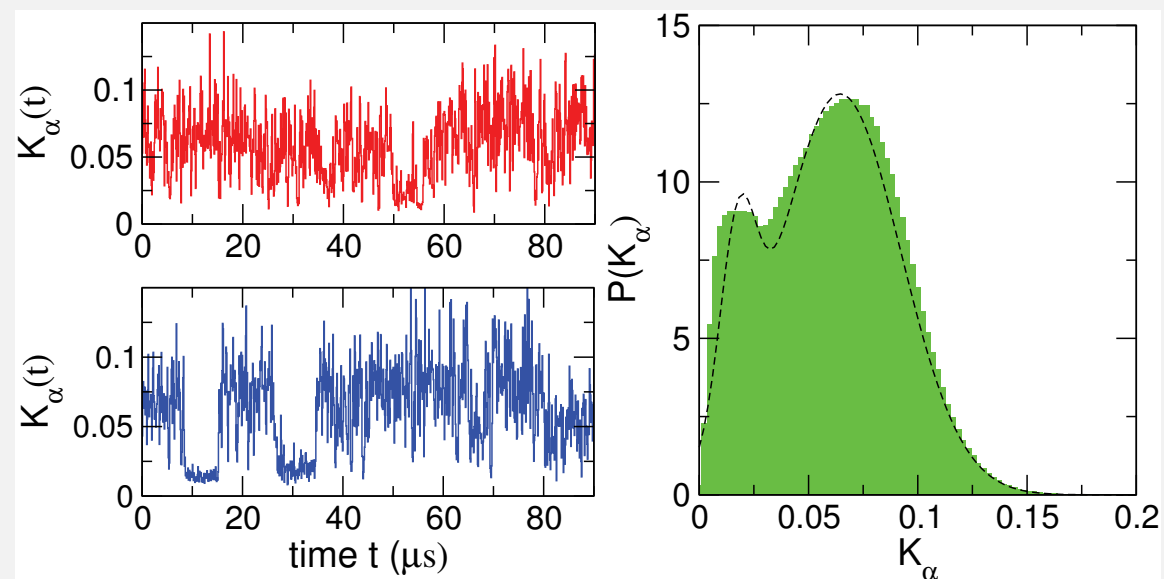


Crowding in membranes increases dynamic heterogeneity



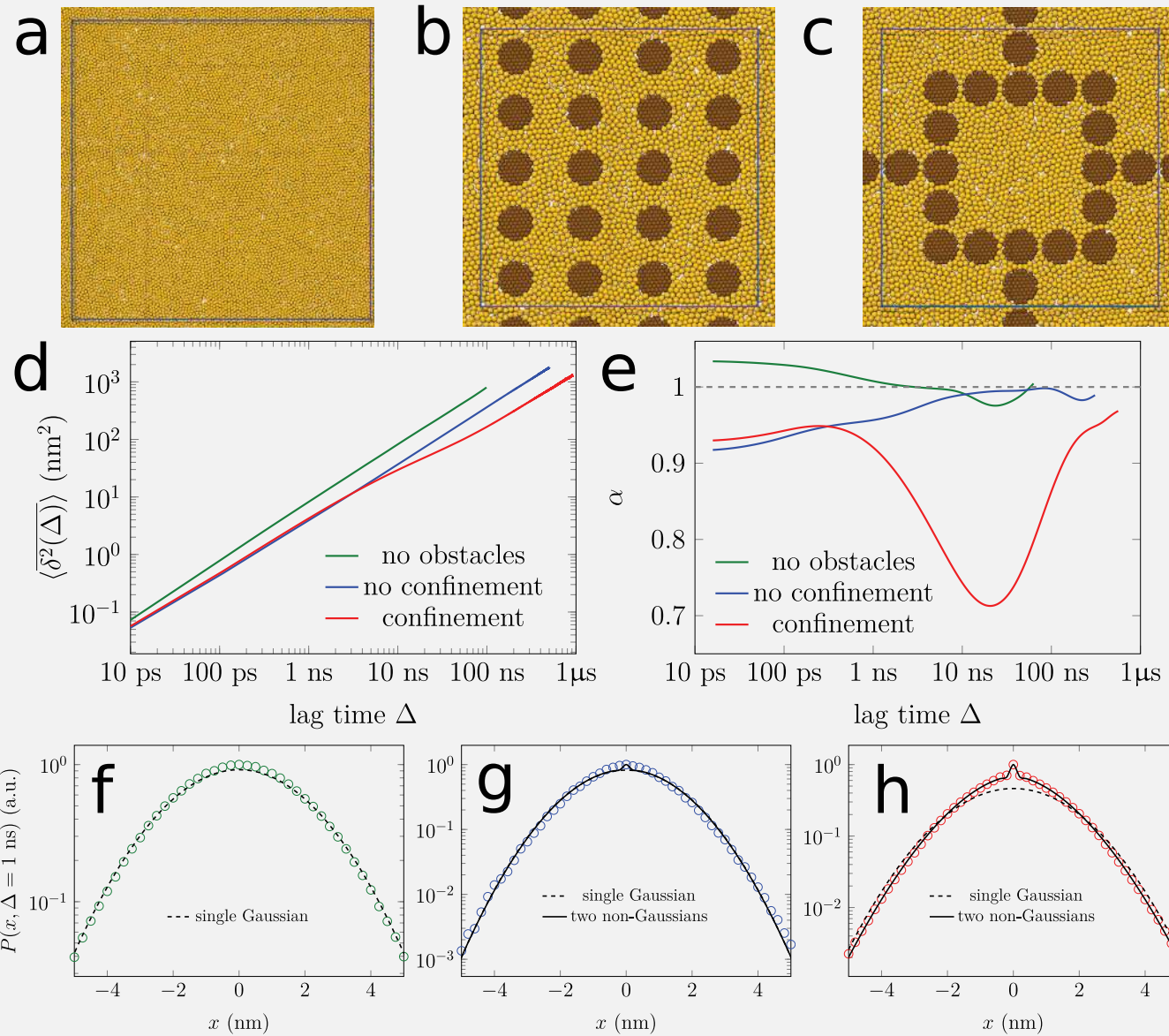
Diffusivity(t) for two lipids

Lipid diffusivity, dilute membrane

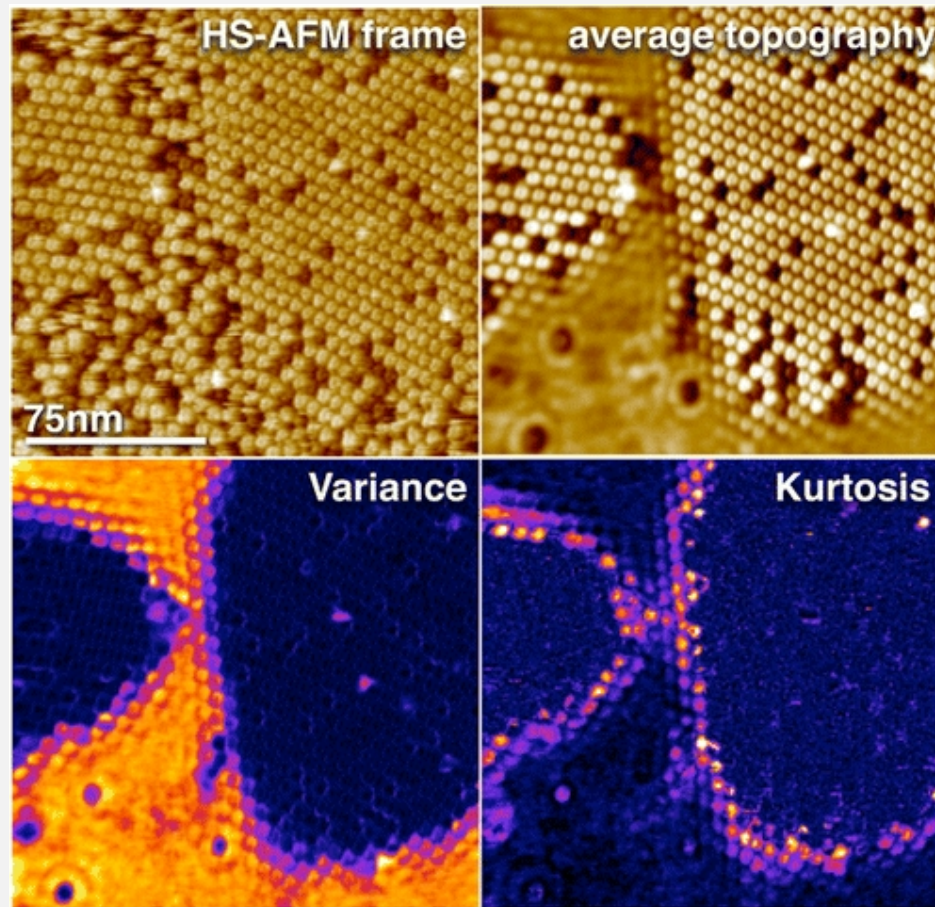


Lipid diffusivity, crowded membrane

Crowding in membranes: confinement in argon system

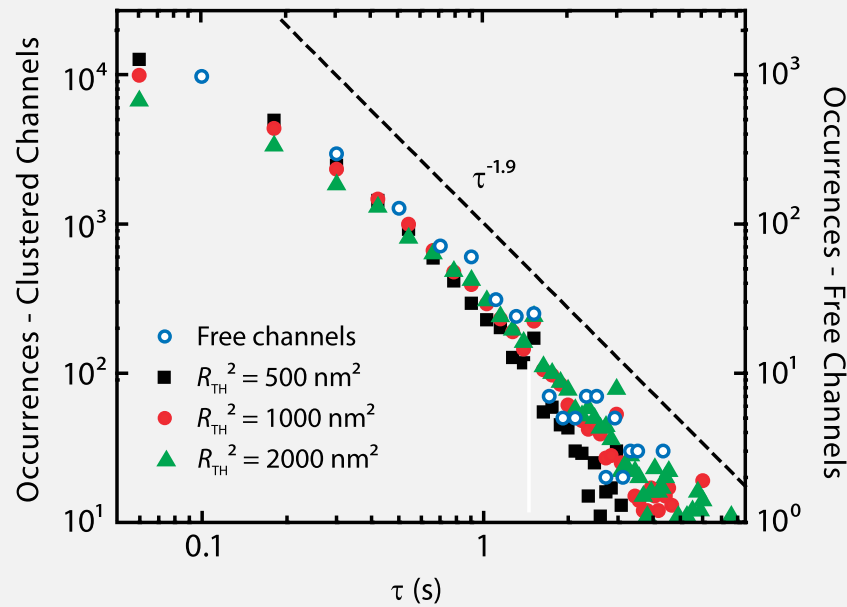
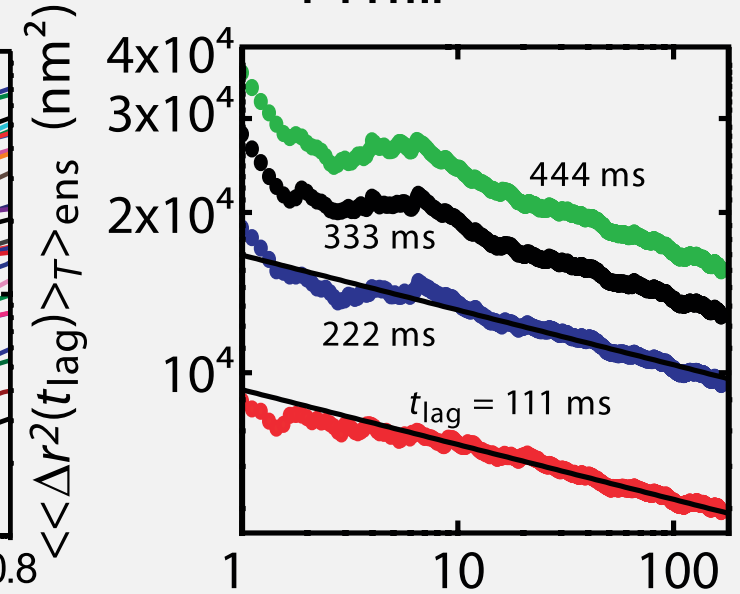
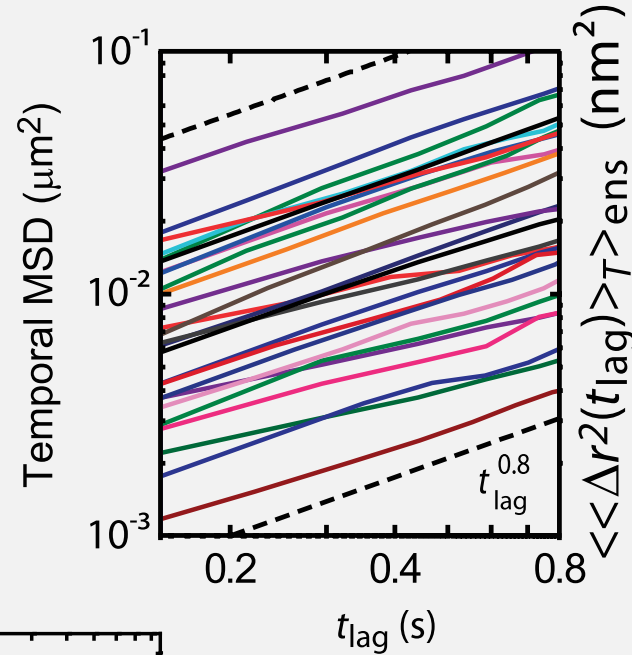
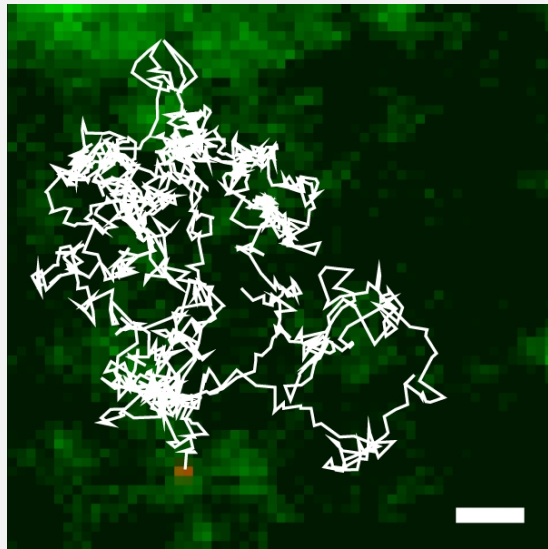


Glass-like protein diffusion in crowded membranes



In vitro high speed AFM of supported lipid bilayers reveals trapped proteins in neighbour-formed cages for time scales up to **10 sec**. Clearly separated areas of different diffusive (in particular, non-Gaussian) diffusion.

CTRW-like motion of Ka channels in plasma membrane

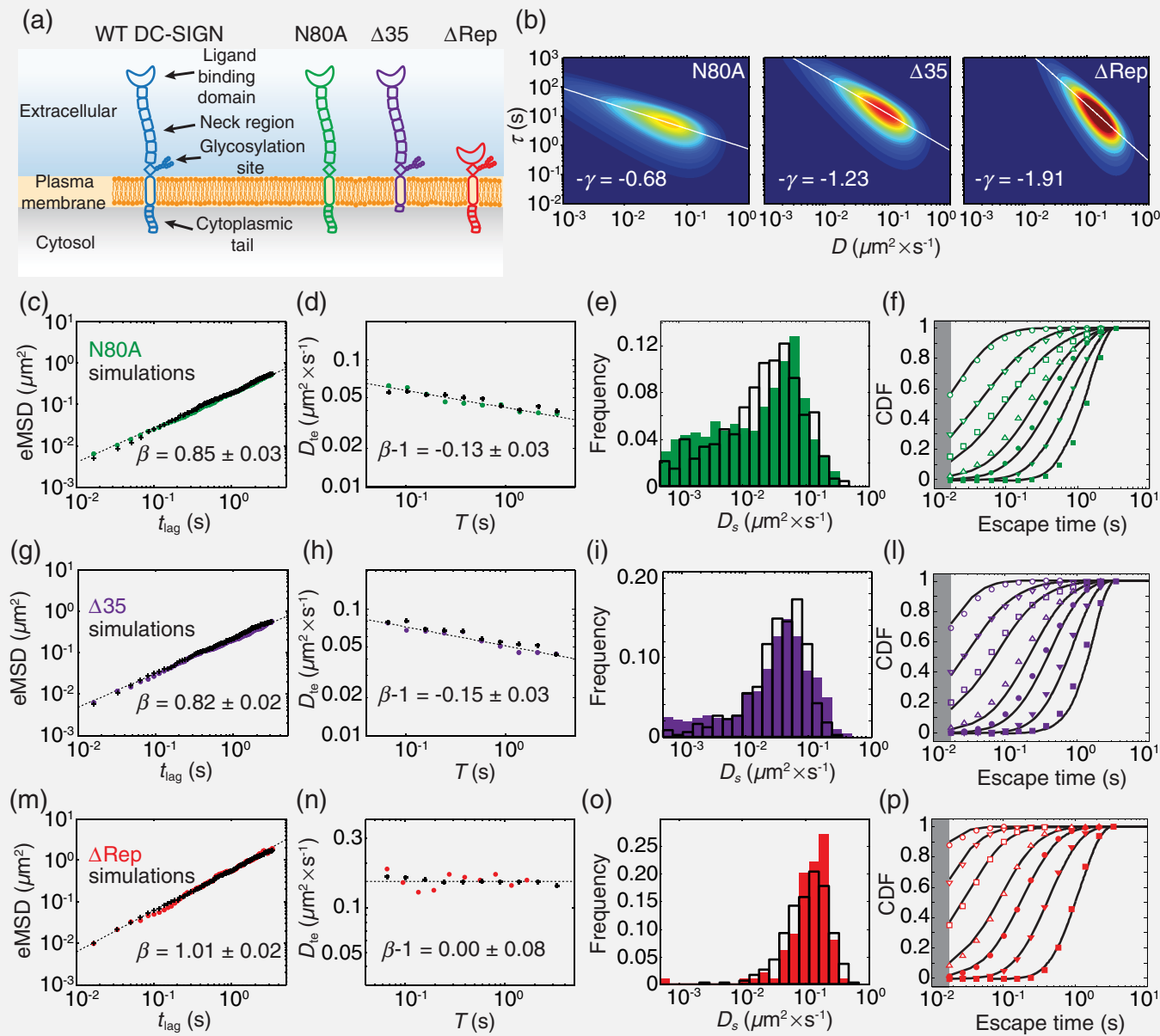


$$\psi(\tau) \simeq \tau^{-1-\alpha} \leadsto \langle \mathbf{r}^2(t) \rangle \simeq t^\alpha$$

$\overline{\delta^2(\Delta)}$ apparently random

$$\overline{\delta^2(\Delta)} \simeq K_\alpha \frac{\Delta}{T^{1-\alpha}}$$

Ageing in the motion of membrane embedded proteins



Time averaged MSD & weak ergodicity breaking (WEB)

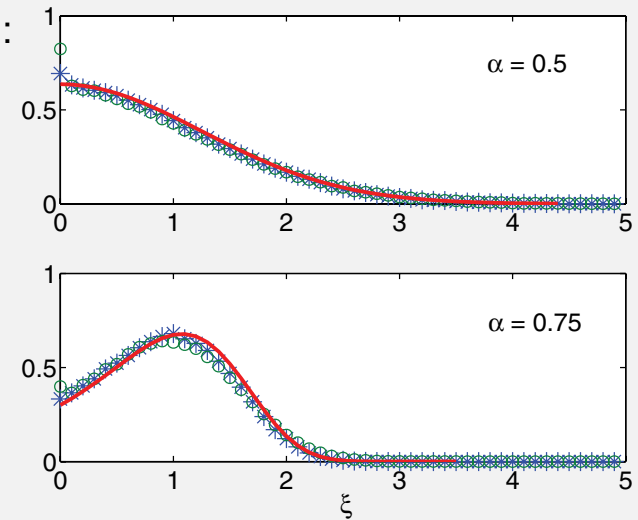
Time averaged MSD $\simeq \Delta$ is pseudo-Brownian and ageing ($\langle x^2(t) \rangle \simeq K_\alpha t^\alpha$):

$$\langle \overline{\delta^2(\Delta)} \rangle \sim \frac{1}{N} \sum_i^N \overline{\delta_i^2(\Delta)} \sim \frac{2dK_\alpha}{\Gamma(1+\alpha)} \frac{\Delta}{T^{1-\alpha}} \quad \therefore \quad K_\alpha \equiv \frac{\langle \delta \mathbf{r}^2 \rangle}{2\tau^\alpha}$$

Amplitude distribution $\overline{\delta^2}$ of trajectories ($\xi \equiv \overline{\delta^2} / \langle \overline{\delta^2} \rangle$):

$$\phi_\alpha(\xi) \sim \frac{\Gamma^{1/\alpha}(1+\alpha)}{\alpha \xi^{1+1/\alpha}} L_\alpha^+ \left(\frac{\Gamma^{1/\alpha}(1+\alpha)}{\xi^{1/\alpha}} \right)$$

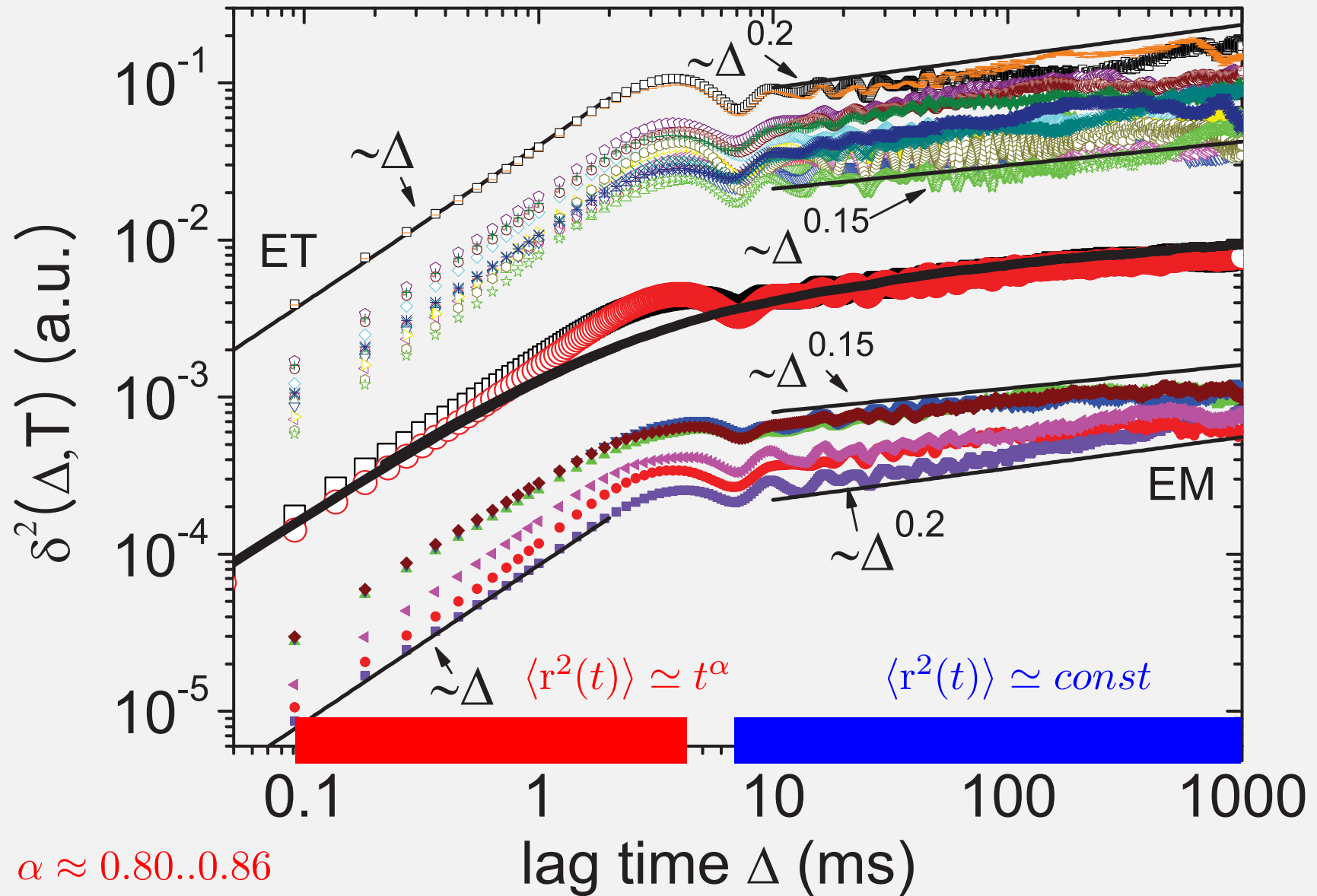
$$\phi_{1/2}(\xi) = \frac{2}{\pi} \exp \left(-\frac{\xi^2}{\pi} \right); \quad \phi_1(\xi) = \delta(\xi - 1)$$



Confinement does not effect a plateau ($\langle x^2(t) \rangle \simeq \text{const}(T)$):

$$\langle \overline{\delta^2(\Delta)} \rangle \sim \left(\langle x^2 \rangle_B - \langle x \rangle_B^2 \right) \frac{2 \sin(\pi\alpha)}{(1-\alpha)\alpha\pi} \left(\frac{\Delta}{T} \right)^{1-\alpha}; \quad \frac{1}{(K_\alpha \lambda_1)^{1/\alpha}} \ll \Delta \ll T$$

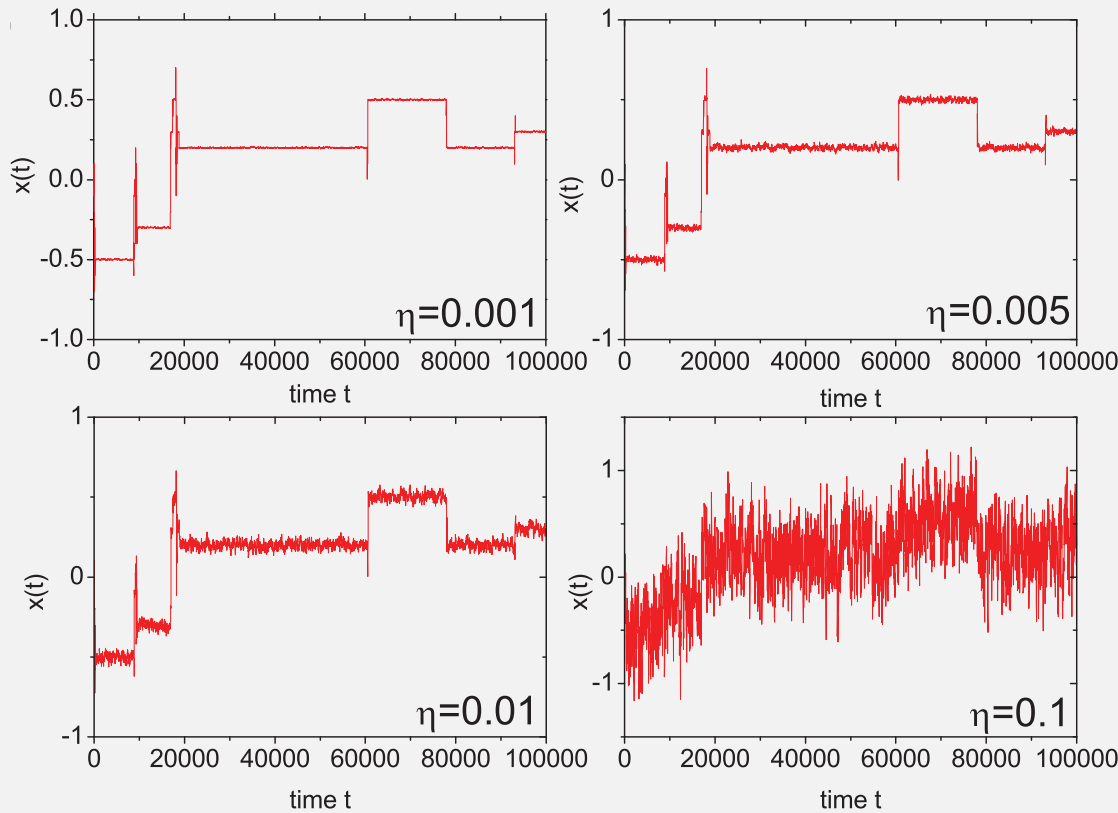
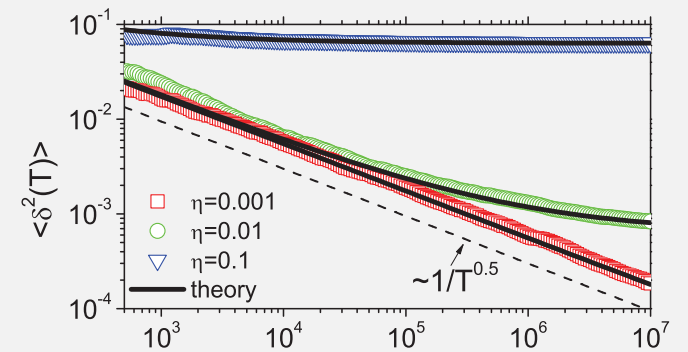
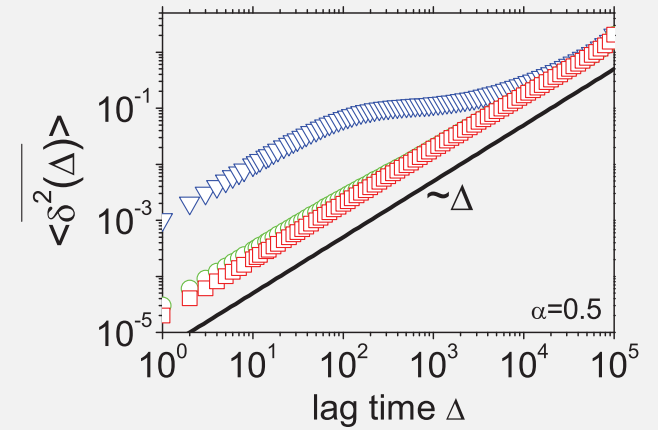
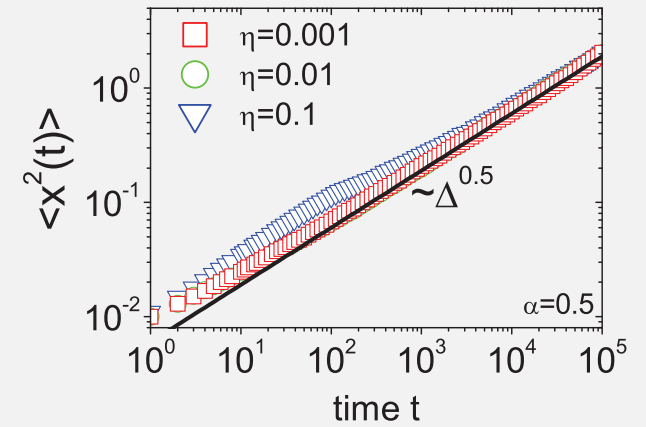
Granule subdiffusion in harmonic optical tweezer potential



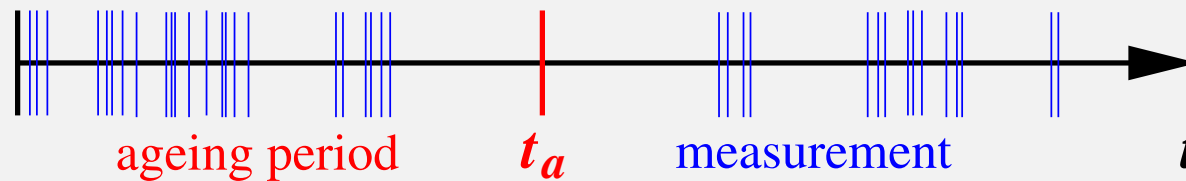
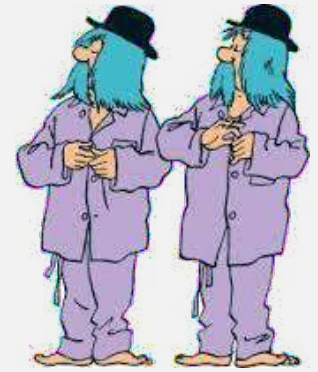
Noisy CTRW processes /w OU-noise

$$\langle x^2(t) \rangle = \frac{2K_\alpha}{\Gamma(1+\alpha)} t^\alpha + \frac{\eta^2 D}{k} (1 - e^{-2kt})$$

$$\langle \overline{\delta^2(\Delta)} \rangle \sim \frac{2K_\alpha}{\Gamma(1+\alpha)} \frac{\Delta}{T^{1-\alpha}} + \frac{2\eta^2 D}{k} (1 - e^{-k\Delta})$$



Ageing effects in single trajectory time averages

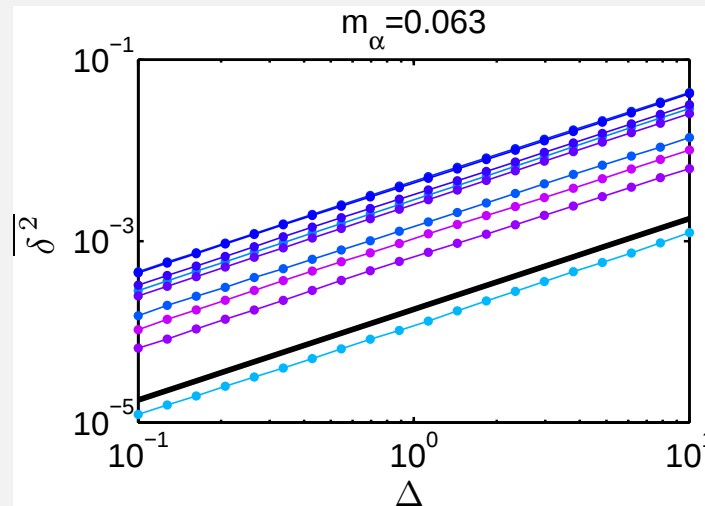
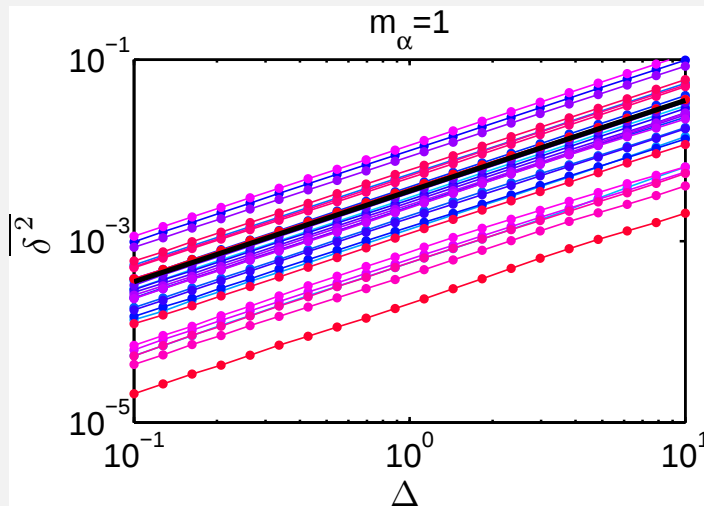


Ageing mean squared displacement ($\Lambda(z) = (1 + z)^\alpha - z^\alpha$)

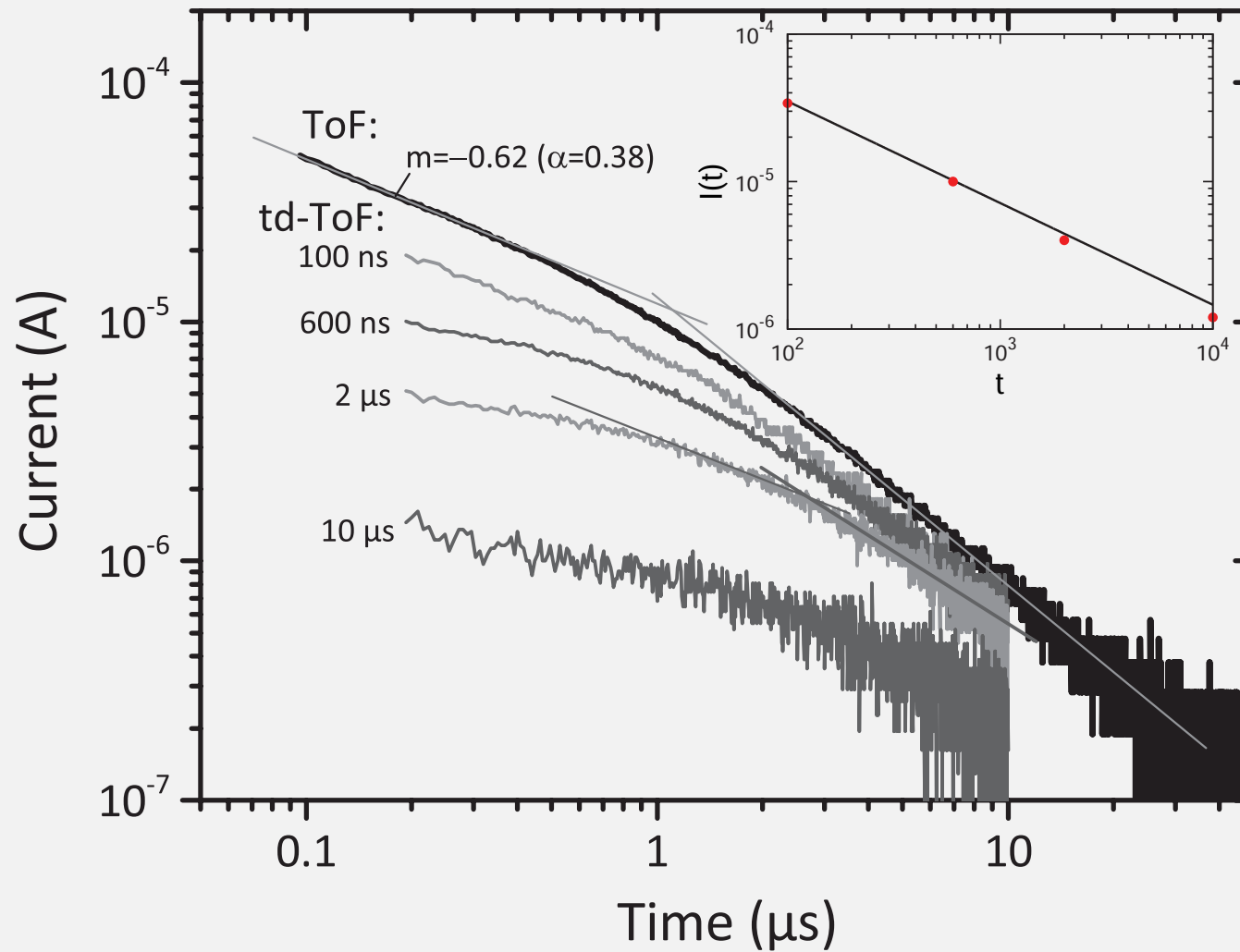
$$\left\langle \overline{\delta^2(\Delta)} \right\rangle_a = \frac{\Lambda_\alpha(t_a/T) g(\Delta)}{\Gamma(1 + \alpha) T^{1-\alpha}} \Leftrightarrow \langle x^2(t) \rangle_a \simeq \begin{cases} t^\alpha, & t_a \ll t \\ t_a^{\alpha-1} t, & t_a \gg t \end{cases}$$

Probability to make at least one step during $[t_a, t_a + T]$: *population splitting*

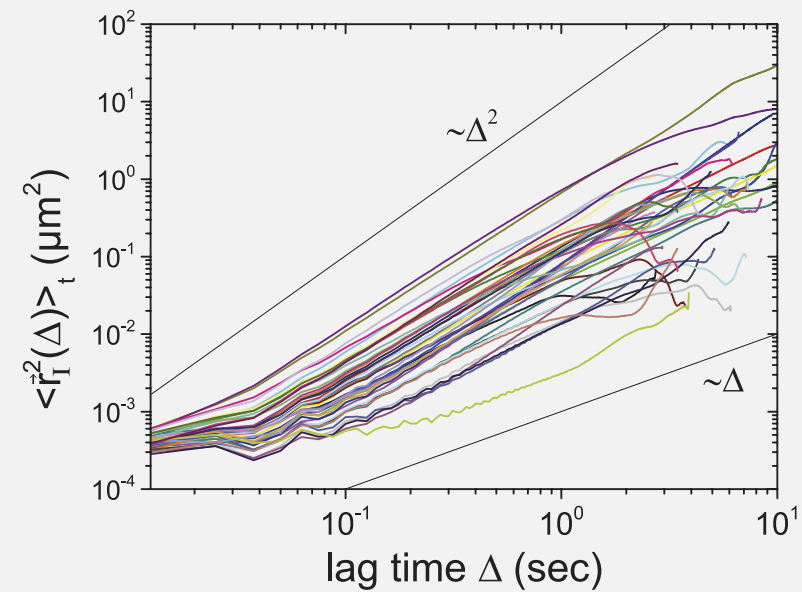
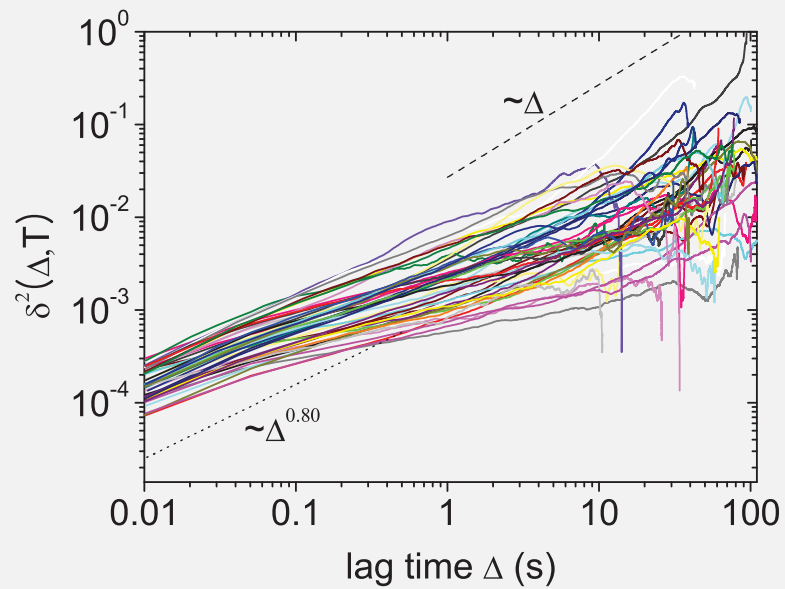
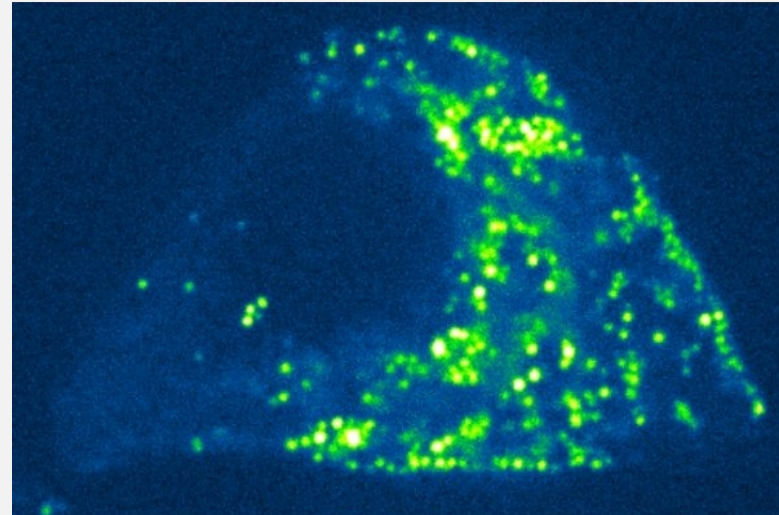
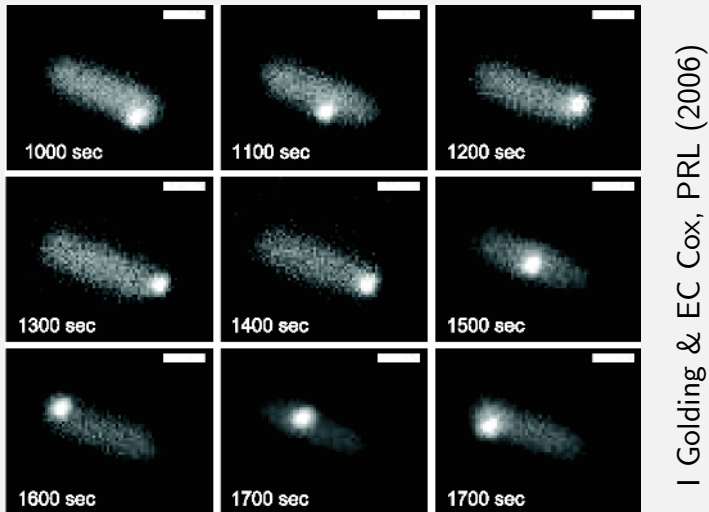
$$m_\alpha(T/t_a) \simeq (T/t_a)^{1-\alpha}, \quad T \ll t_a$$



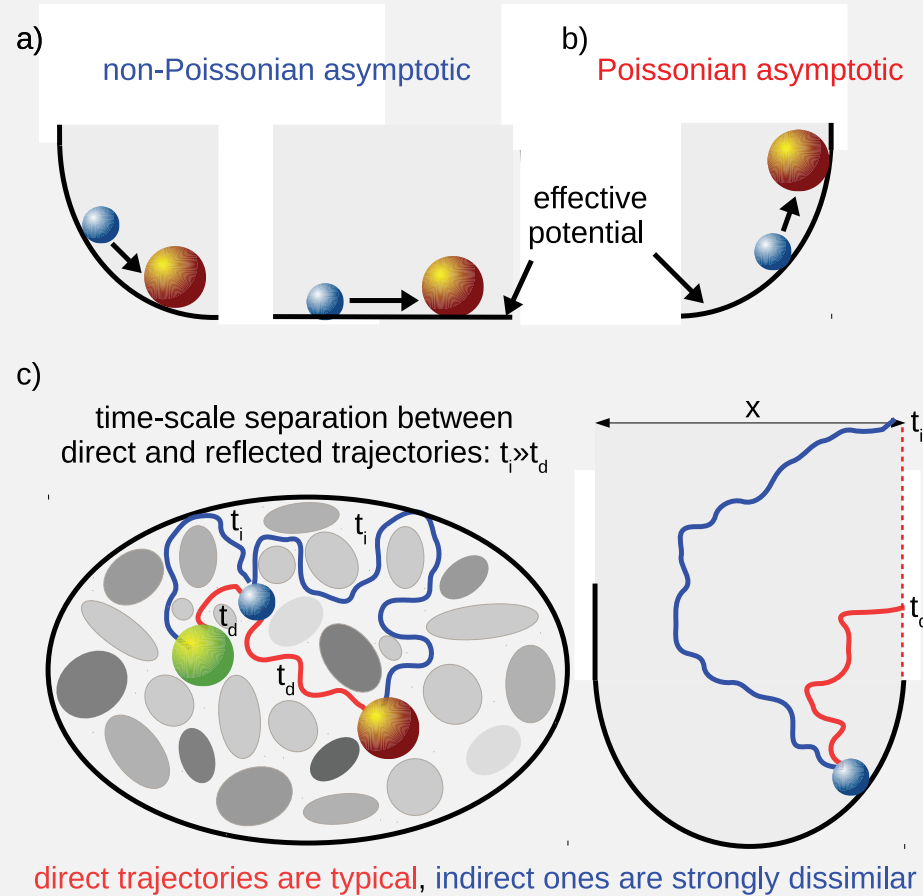
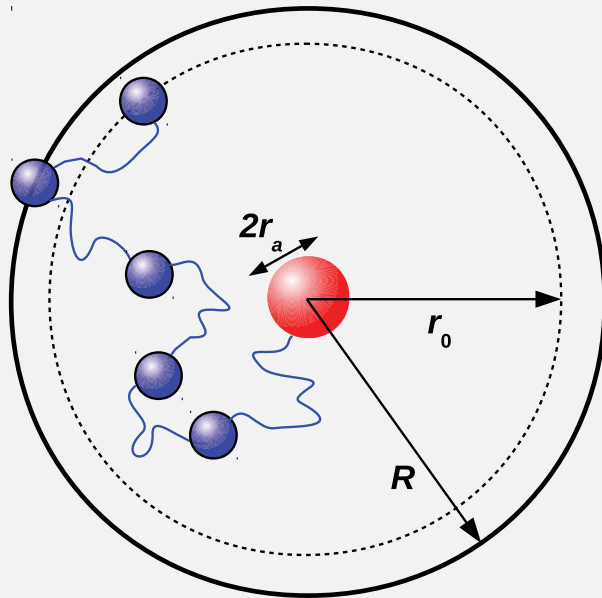
Ageing charge carrier motion in polymeric semiconductors



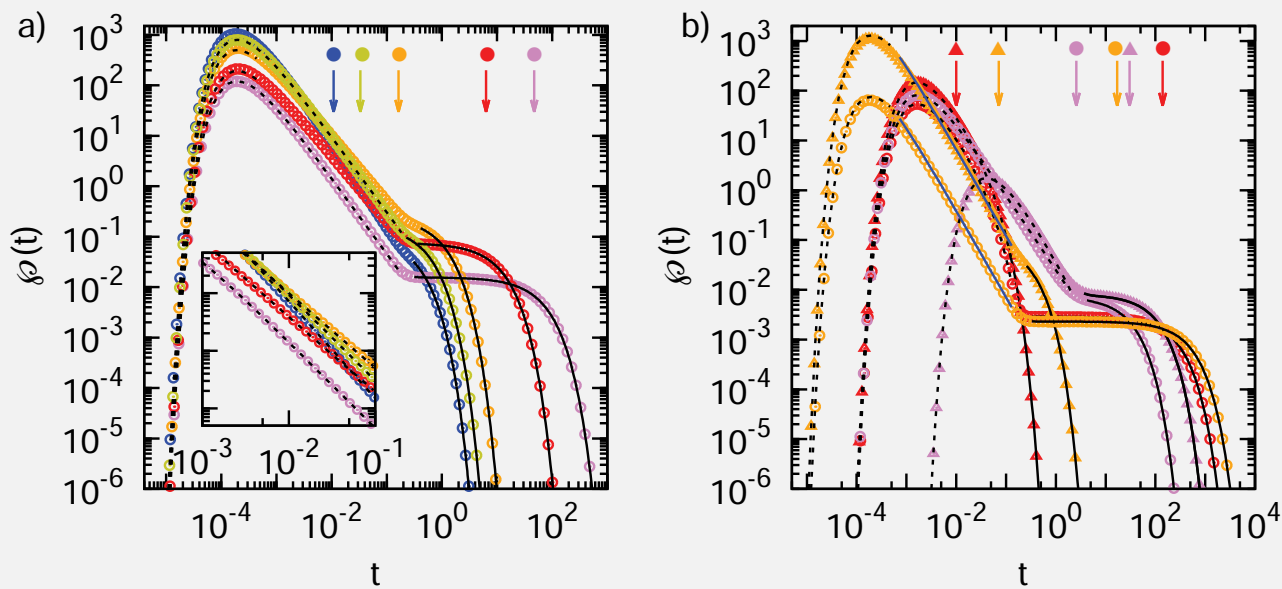
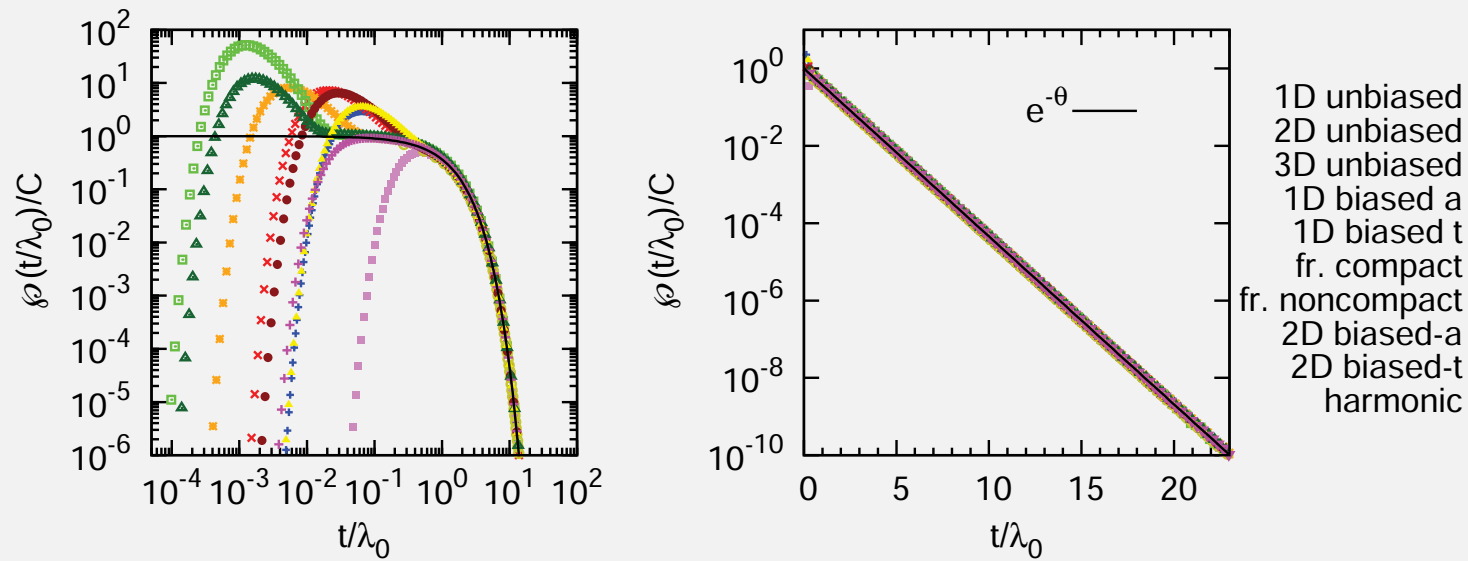
In vivo anomalous diffusion of submicron tracers



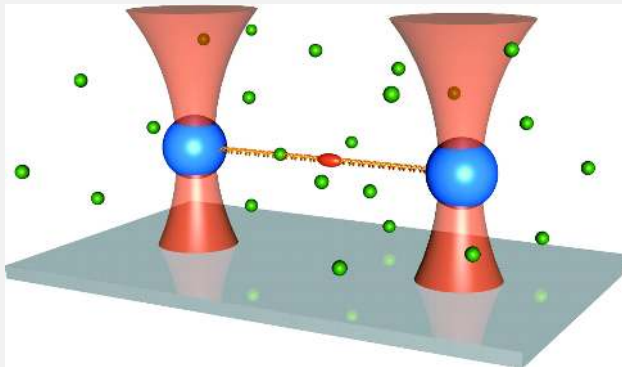
Universal proximity effect in few encounter limit



Universal proximity effect in few encounter limit



Spatiotemporal gene regulation by diffusing regulatory proteins

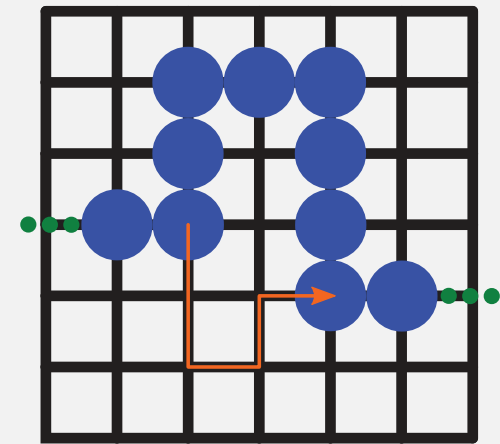


← **DNA conformations**

B van den Broek, S-M Kalisch, MA Lomholt,
RM & GJL Wuite, PNAS (2008)

MA Lomholt, B van den Broek, S-M Kalisch,
GJL Wuite & RM, PNAS (2009)

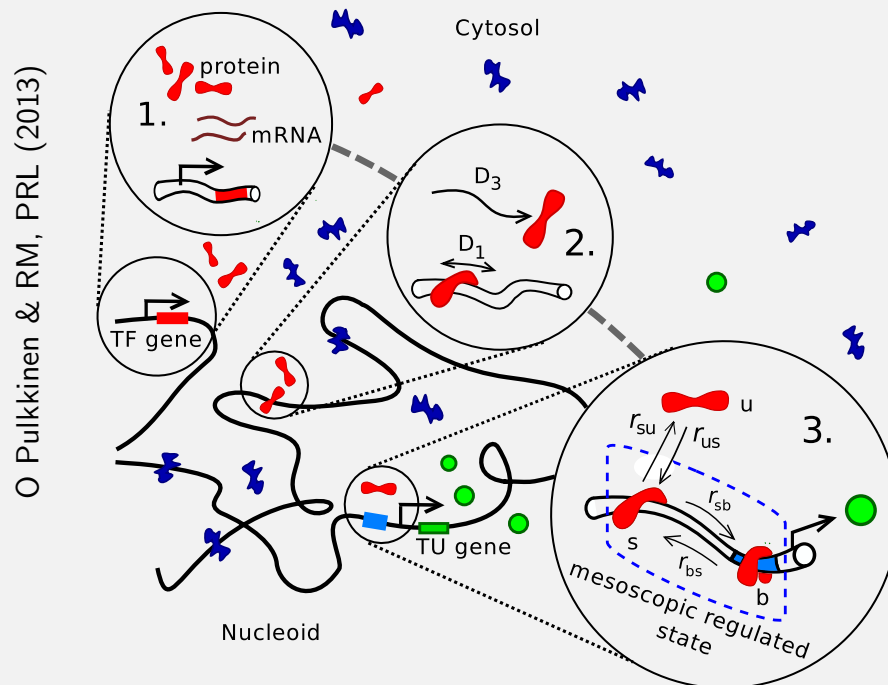
In vivo facilitated diffusion →



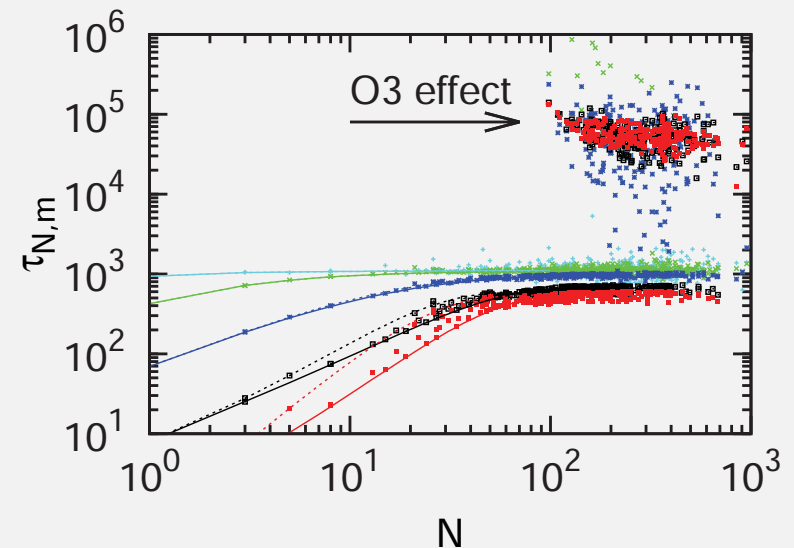
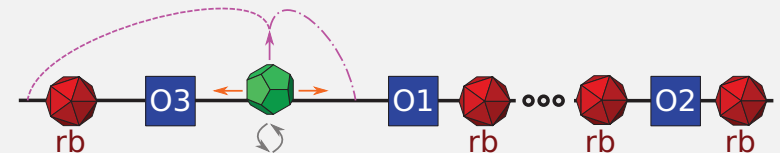
M Bauer & RM, PLoS ONE (2013)

Gene-gene talk: distance matters

Real sequence effects



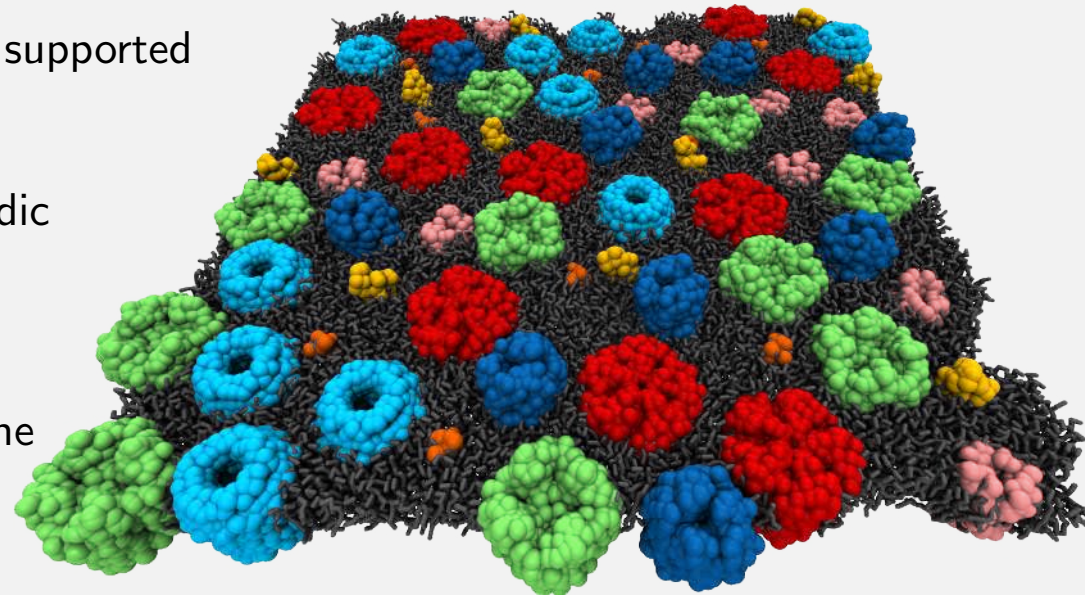
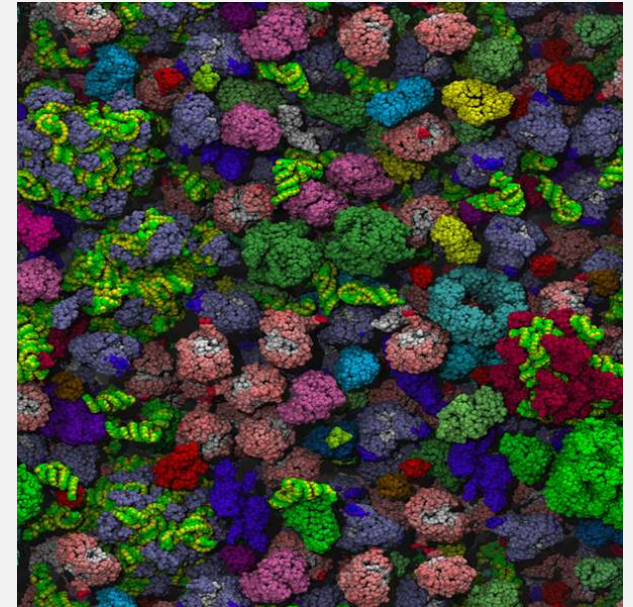
O Pulkkinen & RM, PRL (2013)



M Bauer & RM, Sci Rep (2015)

Σ ummary

- I Transient anomalous diffusion in lipid bilayers:
FBM-type motion
- II Disorder increases anomalies: cholesterol
Also anomaly persistence in gel phase
- III Non-Gaussian, heterogeneous diffusion in crowded bilayers
Features qualitatively reproduced in confined argon
- IIII Glass-like dynamics in crowded in vitro supported
lipid bilayers
- IIII CTRW-like, ageing & weakly non-ergodic
diffusion of protein channels in
plasma membranes
- IIII To analyse data you need to know some
of the tricks: RM & al, PCCP (2014)



Acknowledgements

Vincent Tejedor, Johannes Schulz, **Jae-Hyung Jeon (KIAS→ POSTECH)**

Igor Sokolov (HU Berlin), Irwin Zaid (U Oxford), Otto Pulkkinen (TUT)

Aljaz Godec, Max Bauer, Andrey Cherstvy (U Potsdam)

Michael Lomholt (Syddansk U Odense), Tobias Ambjörnsson (Lunds U)

Ilpo Vattulainen, Hector Martinez-Seara, Matti Javanainen (TUT/U Helsinki)

Marcin Magdziarz (TU Wrocław), Vladimir Palyulin (TUM)

Surya Ghosh (Saclay), Jaeoh Shin (MPI Dresden)

Eli Barkai, Stas Burov, Yuval Garini (Bar-Ilan U)

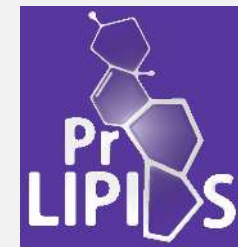
Anna Bodrova (HU Berlin), Gleb Oshanin (Paris)

Aleksei Chechkin (KIPT Kharkov & U Padova)

Henning Krüsemann, Yousof Mardoukhi, Igor Goychuk (U Potsdam)

Christine Selhuber (U Kiel), Kirstine Berg-Sørensen (DTU)

Lene Oddershede (NBI Københavns U)



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