

Numbers, Language, and the Human Mind

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Table of Contents

Introduction

1 Numbers and objects

Cardinal number assignments: '3 pens', '3 kg', '3 °C'

Ordinal number assignments: 'the third runner'

Nominal number assignments: 'bus #3', 'football player #3'

Overview: How to use numbers

2 What does it mean to be a number?

The intersective analysis of numbers

The itemising approach to numbers

The relational view of numbers

A criteria-based view of numbers

3 Can words be numbers?

Counting words are special

Counting words are non-referential

Counting words are acquired differently

A cut with Occam's razor

Counting words as numerical tools

4 The language legacy

Before language: Quantitative capacities in infants and animals

Number sense and beyond

Symbolic reference and the evolution of language

Matching patterns in symbolic reference and number assignments

Language opens the way for numerical cognition: Further support

A possible scenario for the emergence of numerical tools

Language and number as human faculties

5 Children's route to number: From iconic representations to numerical thinking

The acquisition of a counting sequence
Pre-numerical props for number assignments
Gateway to number
Language and the emergence of counting and cardinality

6 The organisation of our cognitive number domain

Representation of counting words as numerical tools
Concepts of numerical quantity
Abstract cardinalities
Concepts of measure
Concepts of numerical rank
Concepts of numerical label
The architecture of the number domain

7 Non-verbal number systems

Arabic numerals as a non-verbal number sequence
The development of non-verbal numerals: From iconic representations to numerical tools
Two alternative number sequences: The correlation of counting words and Arabic numerals

8 Numbers in language: The grammatical integration of numerical tools

Counting words and their referential cousins
How to express numerical concepts: The organisation of number word constructions
Referential number words are not outcasts
The return of the counting words

References

Index

Appendix

Introduction

Numbers play a central role in our lives. Numbers and number expressions arise early both in human history and in the individual development of children. Numerical notations existed long before the invention of script, and archaeological evidence suggests that at least 30,000 years ago our ancestors used notches as primitive representations for collections of things. New-borns in their first week of life can already distinguish two from three objects, and children start using number words and engage in counting games as early as 2 years of age. In our everyday life we use numbers in a wide range of different contexts; we employ them not only for counting, but also for telling the time, on price tags, for football scores, to rank runners in a marathon, for bus lines, as telephone numbers, in lotteries, and so on. And in one way or the other, numbers play a role in the spiritual contexts of most cultures. They are employed in fortune telling, and in many cultures certain numbers, say 13, are associated with bad luck, or certain kinds of number assignments fall under taboo restrictions, for instance, it might be considered imprudent to count people or to count your own children.

What makes numbers such a fascinating topic? I think the fascination that numbers have for us arises from their great significance as reasoning devices, as powerful and highly flexible mental tools. Here is a passage by the 19th/20th century mathematician Richard Dedekind that emphasises the role that this concept of numbers plays in our thinking:

“Of all the devices the human mind has created to make its life - that is, the task of reasoning - easier, there is none that has such a great effect and is so indivisibly connected to its innermost nature, as the concept of *number*. [...] Every thinking human, even if he does not feel it clearly, is a numerical being [...].”

What is it that makes us ‘numerical beings’? What does our concept of number encompass? One aspect that initially comes to mind is cardinality, the property that we ask for in ‘how many’. We assess this property in a cardinal number assignment; for example, I could count the pens that lie on my desk and as a result, assign them the number 3. This kind of number assignment allows us, among others, not only to distinguish two from three objects, but also to acknowledge the difference between, say, 102 and 103 objects. Employing numbers to identify cardinalities enables us to grasp relations far beyond the limits of our perception.

In the context of numbers, one tends to first think of these cardinal number assignments. However, as the examples of bus lines and marathon runners above might have suggested, an important point for our investigation will be that there is more to numbers than cardinality. Numbers, I will argue, are flexible tools that can be used to identify a whole range of properties, and of these properties, cardinality is just one, and crucially not the only one. Apart from cardinality, numbers can be employed to identify, for instance, the ranks of runners (an ordinal number assignment) or the identity of bus lines (a nominal number assignment). Cardinal assignments play an important role in our number concept, though, and in fact our every-day usage of the word ‘number’ often does not distinguish between numbers and cardinality: we use ‘number’ not only when talking about, say ‘the natural numbers’, but also when we want to refer to the cardinality of a set, as for instance in ‘the number of stars’. Since it is crucial for our discussion to distinguish these two meanings, I will use the term ‘number’ only in the first sense, while ‘number’ in the second sense will always be called ‘cardinality’.

Books on numbers and numerical thinking usually concentrate on this cardinal aspect of numbers. The present investigation complements these works by integrating cardinality into a wider view of the number domain. In order to do so, I will distinguish numbers and the properties they can identify in number assignments. Among others, this allows us to take into account not only cardinal number contexts, but also the ordinal and nominal number contexts I mentioned above. What brings together these different contexts? How can we employ the same numbers for such diverse purposes? In this book, I will show that it is a certain pattern of linking up numbers and objects that makes this possible, and I will argue that it is the human language faculty that lays the foundations for this kind of linking. Hence - as I hope to convince you - language plays a crucial role in the emergence of systematic numerical thinking in humans.

Now this will strike you as a bit odd - after all, it was only some paragraphs before that I mentioned that infants can already distinguish three from two objects in their first week of life, hence at a time long before they exercise their language faculty. And what is more, this capacity is not restricted to humans; a lot of other species - among others, monkeys, dolphins, rats, and even canaries - have been shown to discriminate between small sets of different sizes, for instance between two dots and three dots on a display. However, note that this does not mean that they have a full grasp of number. It means that they grasp the different cardinalities of different small sets of dots, that is, they grasp the property of cardinality, but this does not yet involve applying numbers to this property. As

a result, a canary might be able to tell two sunflower seeds from three, but not 102 seeds from 103.

What does it mean, then, to apply numbers? When would we say that somebody has a grasp of numbers? The answer I am going to offer is two-fold. On the one hand, I spell out the crucial properties that a number sequence must have, a list of criteria that help us to identify numbers (I will call this the ‘criteria-based approach to numbers’). On the other hand, I show what it takes to employ numbers in the different kinds of number assignments. In particular, I show that it is not enough to simply link up a number with an object or a set of objects - say, linking up a number with a set of pens, or with a marathon runner, or with a bus line. Rather, for a systematic number assignment we have to link up relations from two different *systems* (a system of numbers and a system of objects). This kind of linking will be called ‘(system-)dependent linking’.

This gives us the basis to identify possible number sequences and to understand what it needs to employ them as numerical tools. And this, then, is the point where language comes in. I will argue that language as a mental capacity allows us to employ dependent links in a systematic way, namely when we correlate words and objects, and I will show how this pattern of dependent linking can be transferred to the numerical domain, where it allows us to link up *numbers* and objects. In this context, counting words - sequences like ‘one, two, three, ...’ as we use them in counting - will play a crucial role as a bridge between language and number in the emergence of systematic numerical cognition. I will show that the counting sequences of natural languages make it possible to use the powerful pattern of dependent linking in the numerical domain. It is this transfer of dependent linking, I will argue, that enables us to develop a unified concept of number, a concept that encompasses cardinal, ordinal, and nominal aspects alike.

Before we now start our investigation, let me give you an idea of how this book is organised. Apart from this introduction, there are eight chapters in the book. The first three chapters introduce the central notions and arguments for our account of numerical thinking. Drawing on these foundations, Chapter 4 investigates the cognitive capacities on which our concept of number can build. Crucially, it is here that I make the case for a linguistic foundation of systematic numerical cognition. The last four chapters apply this approach to different areas that are relevant for our investigation: the acquisition and representation of numbers, the architecture of the cognitive number domain, the status of Arabic numerals, and the linguistic behaviour of number words.

The following paragraphs give a brief overview of the questions we will ask in the different chapters, and how they will contribute to our topic of numbers, language, and the human mind.

Chapter 1: Numbers and objects.

How do we use numbers? What properties of numbers are crucial when we apply them to objects? What have three pens, the third runner, and bus #3 in common?

To find out what a number is, we will ask what we do when we employ numbers and apply them to objects. This chapter introduces a three-fold distinction of number contexts that will feature prominently in our further discussion: cardinal contexts like ‘three pens’ and ‘three litres of wine’, ordinal contexts like ‘the third runner’, and nominal contexts like ‘bus #3’. We analyse the properties of numbers that we make use of in each context, and by doing so, identify the crucial features that numbers need to have. I introduce the notion of dependent linking and show that it is this kind of linking that makes the different kinds of number assignments significant.

Chapter 2: What does it mean to be a number?

How can we account for numbers? What can we learn from the philosophical tradition?

This chapter connects our investigation with the philosophical tradition, and in particular with the controversy on the foundations of mathematics that took place at the end of the 19th and beginning of the 20th century. These logico-mathematical approaches to number were not so much concerned with the particular way in which numbers are represented in the human mind, but set out to define numbers as the primary objects of mathematics. However, I will show that they can contribute to our understanding of the cognitive domain of numbers as a whole, as each of them can account for different concepts related to our numerical cognition. The discussion in this chapter, together with the ‘number characteristics’ from the first chapter, will lead us to a criteria-based account of numbers that will serve as the foundation for our approach to numbers as mental tools.

Chapter 3: Can words be numbers?

What would be a good example of numerical tools? How about counting words?

Equipped with our criteria-based account, we identify counting words of natural languages as numerical tools. I argue that these words - verbal entities - can be employed as elements of a number sequence: they do not refer to numbers, they *serve as* numbers (= they serve as the tools in our number assignments). I show that this approach to counting words can

account for their special status within the grammatical system and in language acquisition. At the same time, it suggests an intimate relationship between the linguistic and the numerical domain that will be further explored in the following chapters.

Chapter 4: The language legacy.

What role does language play for the emergence of numerical thinking? Can canaries count? How could a systematic number concept evolve in the history of our species?

We are now in a position to investigate the role that language plays for the foundation of our numerical thinking. We discuss evidence from animal studies and studies with human babies, suggesting that our number concept can build on pre-linguistic capacities that allow us to grasp those properties of objects that are relevant in the different kinds of number assignments (among others, cardinality). I argue that it is our language faculty that then enables us to grasp the pattern of dependent linking that we identified as the constitutive feature of systematic number assignments. I show how a development that starts from iconic cardinality representations, building on pre-linguistic (and pre-numerical) capacities, can lead to the emergence of a systematic number concept once language comes in and our representations get dissociated from their iconic roots.

Chapter 5: Children's route to number: From iconic representations to numerical thinking.

How do children master systematic number assignments? Why is finger counting so popular? Where does language come in?

In this chapter, our results on numerical thinking and the role of language give us a new perspective on the way children first learn to count. We have a look at the acquisition of counting sequences and discuss how children learn to represent them as numerical tools, taking into account our results from the first chapters on the characteristic features of numbers and the status of counting words. We then investigate how children acquire cardinal number assignments when they learn to count and to tell how many. We find that individual development starts from iconic stages (with finger counting as a prominent example), before dependent linking sets in and with it the emergence of a systematic number assignment - a striking analogy to our evolutionary scenario from Chapter 4. With our previous results on the language/number relationship in mind, we discuss the role of children's linguistic development in this transition.

Chapter 6: The organisation of our cognitive number domain.

What are the different aspects of our number concept, and how do they come together?

We can now integrate the acquisition of counting sequences and cardinal number assignments into a comprehensive view of the cognitive number domain. We discuss the acquisition of different numerical concepts, using our distinction of number contexts from the first chapter as a guideline, and see how the characteristics we identified for each kind of number assignments show up in individual development. In this chapter, we talk about our concept of cardinality and its abstraction in arithmetical reasoning, we talk about measure concepts that cover, among others, properties like weight, temperature, time, distance, and trade value, and we give an account of ordinal and nominal number concepts and their prenumerical underpinnings. We bring together our results in a model for the architecture of our number domain.

Chapter 7: Non-verbal number systems.

Where do Arabic numerals come in? How are non-verbal numerals and counting words linked up?

In this chapter we have a closer look at non-verbal systems like that of the Arabic numerals (1, 2, 3, ...), which are not part of a particular language (say, English) or of an alphabet (for instance the Latin one), but have a status that sets them apart from language and script. We find that once numerical thinking has emerged, such numeral systems can develop as a non-verbal, visual alternative to counting sequences, that is, as an alternative system that meets the number criteria defined in our approach. Our analysis will reveal crucial parallels between these non-verbal and verbal sequences, indicating that our account of numerical tools is independent of a specific modality. We integrate non-verbal numerals into our model of the number domain and show how they can be correlated with counting words, not as an alternative way of spelling number words and also not as an alternative way of naming numbers, but as elements of an independent numerical system.

Chapter 8: Numbers in language: The grammatical integration of numerical tools.

What can the grammar of number word constructions tell us about the relationship between language and number?

We distinguish counting words, which have a special status as numerical tools, and the different kinds of number word constructions that refer to numerical concepts: cardinal constructions ('three pens'; 'three litres of wine'), ordinal constructions ('the third runner'), and '#'-constructions ('bus #3'). I show that the grammatical behaviour of

referential number words can be related to a particular interplay between linguistic and conceptual structures. In particular, we identify syntactic and semantic parallels between the different kinds of number words and certain non-numerical expressions, namely quantifier constructions like ‘few pens’ and ‘many pens’, superlatives like ‘the youngest runner’, and proper nouns like ‘my sister Karen’. I motivate the grammatical properties of number words within a broader model of numerical tools, counting words, and language.

In short, then, this book is about our concept of numbers and its foundations, and it is dedicated to exploring the distinctive way in which numerical cognition is intertwined with the human language faculty. Adopting an interdisciplinary perspective, my argumentation will draw on results from different fields related to cognitive science, among them psychology, linguistics, and the philosophy of mathematics. As I hope to convince you in this book, it is this interdisciplinary perspective that has the potential to give us new insights into the rich and fruitful relationship between numbers and language.

With this in mind, I have taken care not to presuppose a particular specialised background for the discussion, but to develop the concepts we need in a largely non-technical manner, without introducing a lot of formal machinery. Since I do not want to leave you without explicit formal accounts of the analyses I develop, though, the discussion in the main body of the text is complemented by an appendix that provides the respective definitions that formalise our model of numerical cognition.