Workshop
DOGS 2025
March 10 - 13/2025, Friedrich-Schiller-Universität Jena

Program

Monday - March 10th

08.50 - 09.00 **Welcome**
09.00 - 09.50 Yehuda Pinchover - *On existence of minimizers for weighted L^p-Hardy inequalities on $C^{1,\gamma}$-domains with compact boundary*
10.00 - 10.50 Florian Fischer - *p-Rellich Inequalities on Graphs*
10.50 - 11.30 **Coffee Break**
11.30 - 12.20 Ujjal Das - *On the Landis conjecture on graphs*
12.20 - 14.00 **Lunch Break**
14.00 - 14.50 Philipp Bartmann - *Semigroups on Simplicial Complexes and Independence of the ℓ^p-spectrum*
15.00 - 15.50 Ian Zimmermann - *Introduction to criticality theory for nonlinear Dirichlet forms*
15.50 - 16.30 **Coffee Break**
16.30 - 17.20 Simon Puchert - *Nonlinear resistance forms*

Tuesday - March 11th

09.00 - 09.50 Uzy Smilansky - *Graph theory and Scrambling of Quantum Information*
10.00 - 10.50 Christopher Cedzich - *Quasiperiodic CMV matrices and the unitary almost Mathieu operator*
10.50 - 11.30 **Coffee Break**
11.30 - 12.20 Joachim Kerner - *Atypical spectral and transport properties of non-locally finite crystals (and maybe more)*
12.20 - 14.00 **Lunch Break**
14.00 - 14.50 Lior Tenenbaum - *Dynamical and spectral defects in approximations of substitution systems*
15.00 - 15.50 Gilad Sofer - *Johnson-Schwartzman gap labelling for almost-periodic quantum graphs*
15.50 - 16.30 **Coffee Break**
16.30 - 17.20 Nora Doll - *Skew localizer for real index pairings*

Wednesday - March 12th

09.00 - 09.50 Olaf Post - *Distances between operators acting on different Hilbert spaces*
10.00 - 10.50 Melchior Wirth - *Interactions between Markov semigroups on discrete, continuous and noncommutative spaces*
10.50 - 11.30 **Coffee Break**
11.30 - 12.20 Noema Nicolussi - *Jacobians of graphs and degenerating Riemann surfaces*
12.20 - 14.00 **Lunch Break**

14.00 - 14.50 Elias Zimmermann - *Exponential mixing and spherical equipartition for processes on trees*
15.00 - 15.30 Marius Nietschmann - *Multifractality in quantum star graphs*
16.00 - 18.00 **Hike** - *Short hike to Papiermühle via Napoleonstein (if the weather permits)*
18.00 - ??.?? **Conference Dinner** - *Papiermühle, Erfurter Str. 102, 07743 Jena*

Thursday - March 13th

09.00 - 09.50 Jiří Lipovský - *The spectral determinant for second-order elliptic operators on the real line*
10.00 - 10.50 Philipp Gohlke - *(Non-)intrinsic ergodicity of random substitutions*
10.50 - 11.30 **Coffee Break**
11.30 - 12.20 Marius Neubert - *On Weighted Ergodic Theorems and Substitutive Sequences*
12.30 - 13.00 Yannik Thomas - *Beyond Symmetries of the Kohmoto Butterfly*

Abstracts

Philipp Bartmann
(Universität Potsdam)

Semigroups on Simplicial Complexes and Independence of the ℓ^p-spectrum

In this talk we will study semigroups on weighted abstract simplicial complexes generated by different types of Laplace operators. We will discuss when those semigroups extend consistently from ℓ^2 to ℓ^p. We will then show that the spectrum of the respective generators on ℓ^p is independent of p, assuming a volume growth condition with respect to an intrinsic metric.This generalizes a result by Bauer, Hua and Keller on graphs to the setting of simplicial complexes.

Christopher Cedzich
(HHU Düsseldorf)

Quasiperiodic CMV matrices and the unitary almost Mathieu operator

TBA

Ujjal Das
(BCAM - Bilbao)

On the Landis conjecture on graphs

The Landis conjecture concerns the fastest speed at which a non-trivial solution of a Schrödinger equation can decay at infinity. Such results are referred as *unique continuation at infinity*. In this talk, we will discuss our partial affirmative answer to the conjecture for positive Schrödinger operators on graphs. More precisely, we provide a decay criterion that ensures when $\mathcal{H}$-harmonic functions for a positive Schrödinger operator $\mathcal{H}$ with potentials bounded from above by 1 are trivial. We will specifically consider the special cases of $\mathbb{Z}^d$ and regular trees for which we get a explicit decay criterion. Moreover, the fractional analogue of the Landis conjecture on $\mathbb{Z}^d$ will be discussed. Our approach relies on the discrete version of Liouville comparison principle and criticality theory.

Based on joint work with Matthias Keller and Yehuda Pinchover.

Nora Doll
(MLU Halle)
Skew localizer for real index pairings

In this talk index pairings of a projection and a unitary where both, the projection and the unitary fulfill real symmetry relations are considered. For a given combination of symmetries the integer-valued index of the pairing vanishes, but there may be a Z2-index given by the dimension of its kernel, modulo 2. The aim is then to construct a finite-dimensional real skew-adjoint matrix called the skew localizer for these pairings and to show that the Z2-index can be computed as the sign of the Pfaffian of the skew localizer. The main tool to prove the connection of the Z2-index to the sign of the Pfaffian of the skew localizer is the orientation flow of paths of bounded real skew-adjoint Fredholm operators. This orientation flow might be of independent interest and is introduced in the second part of this talk.

Florian Fischer
(Universität Bonn)
p-Rellich inequalities on graphs

In this talk, we will see two ways of obtaining p-Rellich inequalities on graphs from an interplay of a p-Hardy inequality with the Hölder inequality. Both abstract approaches will be visualised on the natural numbers. The results can be seen as a generalisation to the quasi-linear setting of results by Keller, Pinchover and Pogorzelski.

Philipp Gohlke
(Universität Bielefeld)
(Non-)intrinsic ergodicity of random substitutions

Random substitutions are generalisations of substitutons, where letters are mapped randomly and independently to one of a finite set of possible words. This typically gives rise to dynamical systems with a hierarchical structure, mixed spectral type, and a higher complexity than classical substitution systems. In fact, random substitution systems usually have positive entropy.

Among the many ergodic measures, a special role is played by the measures that maximise the entropy - if there is a unique such measure the system is called intrinsically ergodic. We show that for certain random substitution systems, the measures of maximal entropy are precisely those that are invariant under the so called "shuffle group", introduced in previous work of Fokkink-Rust-Salo. This leads to an equivalent criterion for intrinsic ergodicity in terms of an associated Markov chain. Finally, we illustrate the richness of this class by providing an example with several measures of maximal entropy (joint work with A. Mitchell).

Joachim Kerner
(Fernuniversität Hagen)

Atypical spectral and transport properties of non-locally finite crystals (and maybe more)

In the first part of the talk we discuss recent results on Schrödinger operators on periodic graphs which are non-standard in the sense that we allow vertices to have an infinite number of neighbours. It turns out that such non-locally finite graphs exhibit various phenomena which are absent in the locally finite setting: and this is true from a spectral as well as transport point of view. Using some explicit examples, we shall illustrate such new effects in more detail. Quite surprisingly, it turns out that one of the examples provides us with a negative answer to a question raised by Damanik et al. in a recent paper on ballistic transport (this part of talk is based on joint work with O. Post, M. Sabri, and M. Täufer).

If time allows, we shall also quickly discuss spectral comparison results on discrete graphs. In recent years, various authors have derived such comparison results on Euclidean domains and quantum graphs. Our aim is to present a generalization to the discrete setting. Along the way, we also establish a so-called local Weyl law which is of independent interest (the second part of the talk is based on joint work with P. Bifulco and C. Rose).

Jiří Lipovský
(University of Hradec Králové)

The spectral determinant for second-order elliptic operators on the real line

The spectral determinant for an operator with infinitely many eigenvalues is a generalization of the notion of the determinant of a square matrix. First, we introduce the definition of the spectral determinant. Then we remind a classic result by Levit and Smilansky about the determinant of a Schrödinger operator on an interval. With the methods inspired by this result, we show a formula (and outline its proof) for the determinant of a compact perturbation of a Schrödinger operator on the real line that uses the determinant of a non-perturbed operator and certain Wronskians. Finally, we apply it to a harmonic and anharmonic oscillator. The talk will be based on the joint paper with P. Freitas: P. Freitas, J. Lipovský, The spectral determinant for second-order elliptic operators on the real line, Lett. Math. Phys. 114 (2024), 65. [arXiv:2405.03469]

Marius Neubert
(Universität Leipzig)
On Weighted Ergodic Theorems and Substitutive Sequences

In this talk, we explore the connection between substitution dynamical systems and weighted ergodic theorems. We derive a condition which fixed points of substitutions admit good weights for ergodic theorems. The study of weighted ergodic averages goes back to Wiener and Wintner's theorem from 1941, strengthening the classical pointwise ergodic theorem by Birkhoff. In the following, a vital area of research emerged, resulting both in new classes of "good weights" and conditions to determine if a given sequence yields such a good weight. Based on a result from Eisner and Konieczny, we focus on sequences that emerge from primitive substitutions. We introduce the notion of a substitution dynamical system. Using its properties and the theory of correlation measures, we characterize constant-length substitutions that generate null-good weights.

Noema Nicolussi
(TU Graz)
Jacobians of graphs and degenerating Riemann surfaces

In the last decades, there has been an immense interest in understanding what happens to classical objects associated to a smooth Riemann surface (e.g., invariants, interesting metrics, eigenvalues of Laplacians, ...) when the Riemann surface degenerates. One example is the Jacobian of a Riemann surface, a complex torus which allows to recover the Riemann surface itself by means of the Torelli theorem. An analog of the Jacobian for graphs has been introduced and studied in tropical geometry.

The main objective of the talk is to study the Jacobians of degenerating Riemann surfaces and establish their convergence to a suitable graph Jacobian. If time permits, we also discuss the degeneration behavior of the corresponding Abel-Jacobi maps.

Based on joint work with Omid Amini (École Polytechnique)

Marius Nietschmann
(Universität Potsdam)
Multifractality in quantum star graphs

In the field of quantum chaos, much attention has been given to localization and delocalization properties of chaotic dynamical systems. In the physics literature, it has been conjectured that intermediate systems which lie at the transition point between two physical regimes - such as Anderson localization and delocalization -, feature eigenfunctions that exhibit a "multifractal" structure: a self-similarity in a certain scaling regime which cannot be described by a single fractal exponent, but instead requires a continuous spectrum of exponents.

In this talk, I aim to outline first steps towards extending one of the few known mathematical results confirming multifractality in the context of quantum star graphs

Yehuda Pinchover
(Technion Haifa)

On existence of minimizers for weighted L^p-Hardy inequalities on $C^{1,\gamma}$-domains with compact boundary

Let $p \in (1,\infty)$, $\alpha \in \mathbb{R}$, and $\Omega \subsetneq \mathbb{R}^N$ be a $C^{1,\gamma}$-domain with a compact boundary $\partial\Omega$, where $\gamma \in (0,1]$. Denote by $\delta_\Omega(x)$ the distance of a point $x \in \Omega$ to $\partial\Omega$. Let $\widetilde{W}_0^{1,p;\alpha}(\Omega)$ be the closure of $C_c^\infty(\Omega)$ in $\widetilde{W}^{1,p;\alpha}(\Omega)$, where

$$\widetilde{W}^{1,p;\alpha}(\Omega) := \left\{ \varphi \in W^{1,p}_{\mathrm{loc}}(\Omega) \mid \left(\| \, |\nabla \varphi| \, \|^p_{L^p(\Omega;\delta_\Omega^{-\alpha})} + \|\varphi\|^p_{L^p(\Omega;\delta_\Omega^{-(\alpha+p)})} \right) < \infty \right\}.$$

We study the following two variational constants: the *weighted Hardy constant*

$$H_{\alpha,p}(\Omega) := \inf \left\{ \int_\Omega |\nabla\varphi|^p \delta_\Omega^{-\alpha} \mathrm{d}x \;\middle|\; \int_\Omega |\varphi|^p \delta_\Omega^{-(\alpha+p)} \mathrm{d}x = 1, \varphi \in \widetilde{W}_0^{1,p;\alpha}(\Omega) \right\},$$

and the *weighted Hardy constant at infinity*

$$\lambda^\infty_{\alpha,p}(\Omega) := \sup_{K \Subset \Omega} \inf_{W_c^{1,p}(\Omega\setminus K)} \left\{ \int_{\Omega\setminus K} |\nabla\varphi|^p \delta_\Omega^{-\alpha} \mathrm{d}x \;\middle|\; \int_{\Omega\setminus K} |\varphi|^p \delta_\Omega^{-(\alpha+p)} \mathrm{d}x = 1 \right\}.$$

We show that $H_{\alpha,p}(\Omega)$ is attained if and only if the spectral gap $\Gamma_{\alpha,p}(\Omega) := \lambda^\infty_{\alpha,p}(\Omega) - H_{\alpha,p}(\Omega)$ is strictly positive. Moreover, we obtain tight decay estimates for the corresponding minimizers. Furthermore, when Ω is *bounded* and $\alpha + p = 1$, then $\lambda^\infty_{1-p,p}(\Omega) = 0$ (no spectral gap) and the associated operator $-\Delta_{1-p,p}$ is null-critical in Ω with respect to the weight δ_Ω^{-1}, whereas, if $\alpha + p < 1$, then $\lambda^\infty_{\alpha,p}(\Omega) = \left| \frac{\alpha+p-1}{p} \right|^p > 0 = H_{\alpha,p}(\Omega)$ (positive spectral gap) and $-\Delta_{\alpha,p}$ is positive-critical in Ω with respect to the weight $\delta_\Omega^{-(\alpha+p)}$.

This is a joint work with Ujjal Das and Baptiste Devyver.

Olaf Post
(Universität Trier)

Distances between operators acting on different Hilbert spaces

In this talk we will define and compare several distances (or metrics) between operators acting on different (separable) Hilbert spaces. We consider here three main cases of how to measure the distance between two bounded operators: first by taking the distance between their unitary orbits, second by isometric embeddings (this generalises a concept of Weidmann) and third by quasi-unitary equivalence (using a concept of the speaker).

Our main result is that the unitary and isometric distances are equal provided the operators are both self-adjoint and have 0 in their essential spectra. Moreover, the quasi-unitary distance is equivalent (up to a universal constant) with the isometric distance for any pair of bounded operators. The unitary distance gives an upper bound on the Hausdorff distance of their spectrum. If both operators have purely essential spectrum, then the unitary distance equals the Hausdorff distance of their spectra. Using a finer spectral distance respecting multiplicity of discrete eigenvalues, this spectral distance equals the unitary distance also for operators with essential and discrete spectrum. In particular, all operator distances mentioned above are equal to this spectral distance resp. controlled by it in the quasi-unitary case for self-adjoint operators with 0 in the essential spectrum. We also show that our results are sharp by presenting various (counter-)examples.

Joint work with Sebastian Zimmer.

Simon Puchert
(FSU Jena)

Nonlinear resistance forms

The concept of resistance forms, originating from (bilinear) Dirichlet forms on graphs, has been extended many times, but only to homogeneous settings. In this talk, we introduce nonlinear resistance forms as a much more general setting similar to nonlinear Dirichlet forms.

From a physics standpoint, we would also want to have a notion of resistance metric that is additive over serial circuits. In homogeneous settings, this leads to an appropriate power of the standard resistance. Proving the triangle inequality for these metrics is often quite involved. We will present a resistance metric for arbitrary nonlinear resistance forms that has the desired additivity and admits a rather simple proof of the triangle inequality.

The talk is based on joint work with Marcel Schmidt.

Uzy Smilansky
(Weizmann Institute Rechovot)

Graph theory and Scrambling of Quantum Information

The introduction of quantum computing brought with it the need to adopt classical and well - founded concepts from Information theory to the Quantum Mechanical world. In particular, concepts like Information Scrambling, Chaos and its measure in terms of Lyapunov exponents had to be reformulated to coexist with tenets of Quantum theory such as Heisenberg's uncertainty principle, while complying with Bohr's correspondence principle. I will try to show how a new approach based on many results and concepts from Graph Theory is able to offer a solution to the above mentioned difficulties.

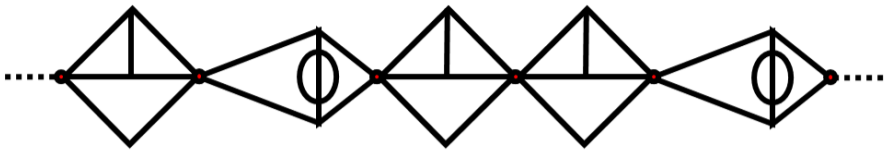
Gilad Sofer
(Technion Haifa)

Johnson-Schwartzman gap labelling for almost-periodic quantum graphs

Given a Schrödinger operator, we are often interested in its integrated density of states, which roughly measures the number of states per unit volume in the system below a given energy. Special attention is often given to the values the integrated density of states attains at spectral gaps, known as *gap labels*, which are of physical importance. For instance, in the integer quantum Hall effect, the gap labels correspond to the quantized values of the Hall conductance. However, predicting these gap labels often requires the use of complicated machinery, such as K-theory, making the proofs quite challenging.

In this talk, we present a more accessible approach to developing gap labelling theorems, based on a method developed by Johnson and Moser. This is done by computing the average winding number of the Prüfer angle for the associated generalized eigenfunctions. Mainly, we present a gap labelling theorem for almost-periodic quantum graphs, providing a simple way to predict the possible gap labels of Sturmian metric graphs and other almost-periodic graphs inspired by one-dimensional tilings. This is done using a concept from dynamical systems known as the Schwartzman group.

Based on joint work in progress with Ram Band.



Lior Tenenbaum
(Technion Haifa)

Dynamical and spectral defects in approximations of substitution systems

The study of Schrödinger operators arising from crystals is a rich and well developed theory. When turning higher dimensional Schrödinger operators of quasicrystals, one often looks for suitable periodic approximations. Recent results by [1] and [3], show that when the underlying dynamical systems converge, the associated spectra converge to the limiting spectrum in the Hausdorff distance.

In a recent work [2] with Ram Band, Siegfried Beckus and Felix Felix Pogorzelski we described a scheme to approximate aperiodic substitution subshifts. In [2] we gave algorithmic criteria for convergence and rate of convergence. In a follow-up work in progress, we elaborate on cases when the scheme fails to yield a successful approximation. We elaborate on the partial limits of the failed approximations and the defect which arise. In

particular, we show that natural approximation candidates for the table tiling substitution yield worm-like impurities and spectral pollution in the essential spectrum. Moreover, we show that certain potential functions, yield spectral pollution of positive measure.

This talk is based on joint work with Ram Band, Siegfried Beckus and Felix Felix Pogorzelski.

References

[1] S. Beckus, J. Bellissard and H. Cornean, *Hölder continuity of the spectra for aperiodic Hamiltonians.*, Ann. Henri Poincaré, 20(11):360–3631, 2019.

[2] R. Band, S. Beckus, F. Pogorzelski and L. Tenenbaum, *Spectral approximation for substitution systems.*, arXiv preprint arXiv:2408.09282, 2024.

[3] RS. Beckus and A. Takase, *Spectral estimates of dynamically-defined and amenable operator families.*, to appear in J. Spectr. Theory, arXiv:2110.05763.

Yannik Thomas

(Universität Potsdam)

Beyond Symmetries of the Kohmoto Butterfly

In this talk, we explore some geometric features of the Kohmoto butterfly, the spectral phase diagram for one-dimensional quasicrystals with Sturmian potentials. We focus on describing properties such as symmetry and spectral defects, as well as how they can be used to study the fractal nature of the Kohmoto butterfly.

This talk is based on an ongoing joint project with Ram Band and Siegfried Beckus.

Melchior Wirth

(Universität Leipzig)

Interactions between Markov semigroups on discrete, continuous and noncommutative spaces

Graph Laplacians, Lindbladians and Laplace-Beltrami operators are all examples of generators of (quantum) Markov semigroups. There is a rich interplay between these different models, for example in the form of group transference which allows to embed certain Lindbladians into Laplace-Beltrami operators acting on matrix-valued functions on Lie groups. I will give an overview how these interactions can be made fruitful in the study of the long-term behavior of the associated semigroups, more precisely for Poincaré and modified logarithmic Sobolev inequalities. This talk is partly based on joint work with Florentin Münch and Haonan Zhang.

Elias Zimmermann
(Universität Leipzig)

Exponential mixing and spherical equipartition for processes on trees

Let $G = (V, E)$ be a Cayley graph. We say that a stochastic process $(\sigma_v)_{v \in V}$ taking values in a finite set satisfies the asymptotic equipartition property (AEP) along a sequence $(F_n)_{n=1}^{\infty}$ of finite subsets $F_n \subseteq V$ if the distribution of the processes $(\sigma_v)_{v \in F_n}$ becomes closer and closer to equipartition as n goes to ∞. For stationary and ergodic processes on $\mathbb{Z}$ the AEP is a consequence of the famous Shannon-McMillan-Breiman (SMB) theorem. Initiated by groundbreaking work of Ornstein and Weiss the SMB theorem has been extensively generalized to graphs with amenable geometry yielding the AEP along suitable sequences of Følner sets such as cubes in $\mathbb{Z}^d$ or balls in Cayley graphs of polynomial volume growth.

However, for Cayley graphs with a hyperbolic geometry only few results have been obtained on equipartition so far. For instance, Berger and Ye could establish the AEP along balls for automorphism invariant and ergodic processes on regular trees, which arise as the Cayley graphs of free groups or free products of cyclic groups. Moreover, recent results of Nevo and Pogorzelski, which utilize the fact that those groups act amenably on their boundary, give rise to horospherical and geodesic equipartition theorems in this setting. In this talk I will explain how one can build on the latter approach and use a quantitative mixing condition to obtain equipartition along spheres in regular trees. Examples of appropriately mixing processes include Gibbs states of Ising models at high temperatures and Potts models with many spin states. The talk is based on joint work with Felix Pogorzelski.

Ian Zimmermann
(FSU Jena)

Introduction to criticality theory for nonlinear Dirichlet forms

We provide an introduction to some aspects of a criticality theory for nonlinear Dirichlet forms. In particular, we give a definition for subcriticality and criticality in this nonlinear setting and discuss some characterizations, some of which involve (weak) Hardy inequalities.

The talk is based on joint work with Marcel Schmidt.