Solution

1. The solutions are

   (a) $\frac{1}{x}$

   (b) $\cos(x)$

   (c) $-x \cdot \sin(x \cdot y)$

   (d) $\frac{1}{1+t^2}$

2. The solutions are

   (a) $\frac{1}{3}x^3 + c$

   (b) $x \ln(x) - x + c$

   (c) 0

   (d) $-\frac{1}{2}\cos^2(x)+c$ (other variants possible)

3. (a) The result of Gaussian Elimination is

$$\begin{array}{cccl} 15 & 20 & 5 & |5 \\ 0 & 17 & 17 & |5 \\ 0 & 0 & 0 & |0 \end{array}$$

   (b) The system has infinitely many solutions with one degree of freedom.

   (c) The solutions can be parameterized as such

$$L = \left\{ \frac{1}{17}\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, t \in \mathbb{R} \right\}$$

   or

$$t \in \mathbb{R}, \quad y = \frac{4}{17} - t, \quad z = t + \frac{1}{17}$$

4. Using a tree diagram or the multivariate hypergeometric distribution, we get

   (a) immediately:

$$\frac{\binom{2}{1}\left(\binom{5}{0}\binom{8}{2} + \binom{5}{1}\binom{8}{1} + \binom{5}{2}\binom{8}{0}\right)}{\binom{15}{3}} = \frac{12}{35}$$

   (b) With a direct computation:

$$\frac{\binom{2}{1}\binom{5}{1}\binom{8}{1}}{\binom{15}{3}} = \frac{16}{91}$$

   (c) Since the probabilities do not depend on the order in which is drawn, we get by conditional probability or path rule:

$$2 \cdot \frac{8}{15}\frac{2}{14} = \frac{16}{105}$$

5. Using conditional expectation with resp. to N:

$$\mathbb{E}[Y] = \mathbb{E}\left[\mathbb{E}\left[\sum_{k=1}^{N} |N\right]\right] = \mathbb{E}[Np] = \lambda p$$

6. By transformation formula, $\frac{X}{X+Y}$ is a uniform random variable on $[0,1]$. Thus

$$\mathbb{E}\left[\frac{X}{X+Y}\right] = \frac{1}{2}$$

7. The limit is 0.
   Hint. Use the rule of Bernoulli and L'Hospital.

8. The requested sets are

   (a) $M \cap N = \{c, e\}$,

   (b) $M \cup N = \{i, c, e, r, a, m\}$,

(c) $M \setminus N = \{i\}$,

(d) $N \setminus M = \{r, a, m\}$,

(e) $M \times N = \{(i,c), (i,r), (i,e), (i,a), (i,m), (c,c), (c,r), (c,e), (c,a), (c,m), (e,c), (e,r), (e,e), (e,a), (e,m)\}$,

(f) $\{\emptyset, \{i\}, \{c\}, \{e\}, \{i,c\}, \{i,e\}, \{c,e\}, \{i,c,e\}\}$.

9. The solutions are:

   (a) 1024

   (b) either

   ```
   int x = 1;
   int i = 0;
   while (i < 10) {
         x *= 2;
         i++;
   }
   ```

   or

   ```
   x = 1
   i = 0
   while (x < 10):
         x *= 2
         i += 1
   ```

10. A sample solution in Python

    (a) using recursion is

    ```
    def f(n):
          if (n==0): return 1
          else: return 3 * f(n-1) - 1
    ```

    (b) and using iteration is

    ```
    def f(n):
          result = 1
          while (n > 0):
                result = 3 * result - 1
                n = n - 1
          return result
    ```