Selftest DATA SCIENCE

1. Compute the following derivatives

   (a) $\frac{d}{dx}\ln(x)$

   (b) $\frac{d}{dx}\sin(x)$

   (c) $\frac{d}{dy}\cos(x \cdot y)$

   (d) $\frac{d}{dt}\arctan(t)$

2. Compute the following integrals

   (a) $\int x^2 dx$

   (b) $\int \ln(x) dx$

   (c) $\int_{-\pi/2}^{\pi/2} \sin(x) dx$

   (d) $\int \sin(x)\cos(x) dx$

3. Look at the following set of linear equations:

$$\begin{aligned} 3x + 4y + z &= 1 \\ 5x + y - 4z &= 0 \\ 6x + 8y + 2z &= 2 \end{aligned}$$

   (a) Use Gaussian Elimination to compute the reduced row echelon form.

   (b) How many solutions does this system have?

   (c) Compute all solutions.

4. There are 15 marbles in a bowl. 5 of them are green, 8 blue and the remaining ones are red. Suppose we draw 3 marbles without replacement.

   (a) What is the probability to draw exactly one red marble?

   (b) What is the probability to draw 3 marbles such that there is one marble of each color?

   (c) What is the probability to draw a red marble after drawing a blue one?

5. Suppose N is a Poisson random variable with parameter λ. For every outcome of N let there be a sequence of independent random variables $X_1, X_2, \ldots, X_N$. Each of the X_k is a Bernoulli variable with parameter p and independent of N. We define

$$Y := \sum_{k=1}^{N} X_k$$

   as the (random) sum of the X_k. Use conditional expectation to compute $\mathbb{E}[Y]$.

6. Let X and Y be independent exponential random variables with the same parameter λ. Compute

$$\mathbb{E}\left[\frac{X}{X+Y}\right]$$

7. Compute the limit

$$\lim_{n\to\infty} \frac{n \log_2 n}{3n^2 + 4n + 1}$$

8. Let $M = \{i, c, e\}$ and $N = \{c, r, e, a, m\}$. Indicate the sets

   (a) $M \cap N$ (set intersection),

   (b) $M \cup N$ (set union)

   (c) $M \setminus N$ (set difference),

   (d) $N \setminus M$ (set difference),

   (e) $N \times M$ (Cartesian product),

   (f) the powerset of M.

9. Consider the following fragment of a program:

```
int x = 1;
for(int i=0; i<10; i++)
     x *= 2;
```

   or, equivalently, in Python syntax:

```
x = 1
for i in range(0,10):
     x *= 2
```

   (a) What is the value of `x` when the loop has left?

   (b) Write down an equivalent `while`-loop. Choose one of the syntactic variants in which the `for`-loop is given.

10. Write down the definitions of a function (in any widely used programming language such as in Python) that computes the following recursively defined function $f(n)$, for all non-negative integers n,

    (a) in a recursive way,

    (b) in an iterative (non-recursive) way.

$$\begin{aligned} f(0) &= 1 \\ f(n) &= 3f(n-1) - 1 \end{aligned}$$