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OF THE EUROPEAN MATHEMATICAL SOCIETY

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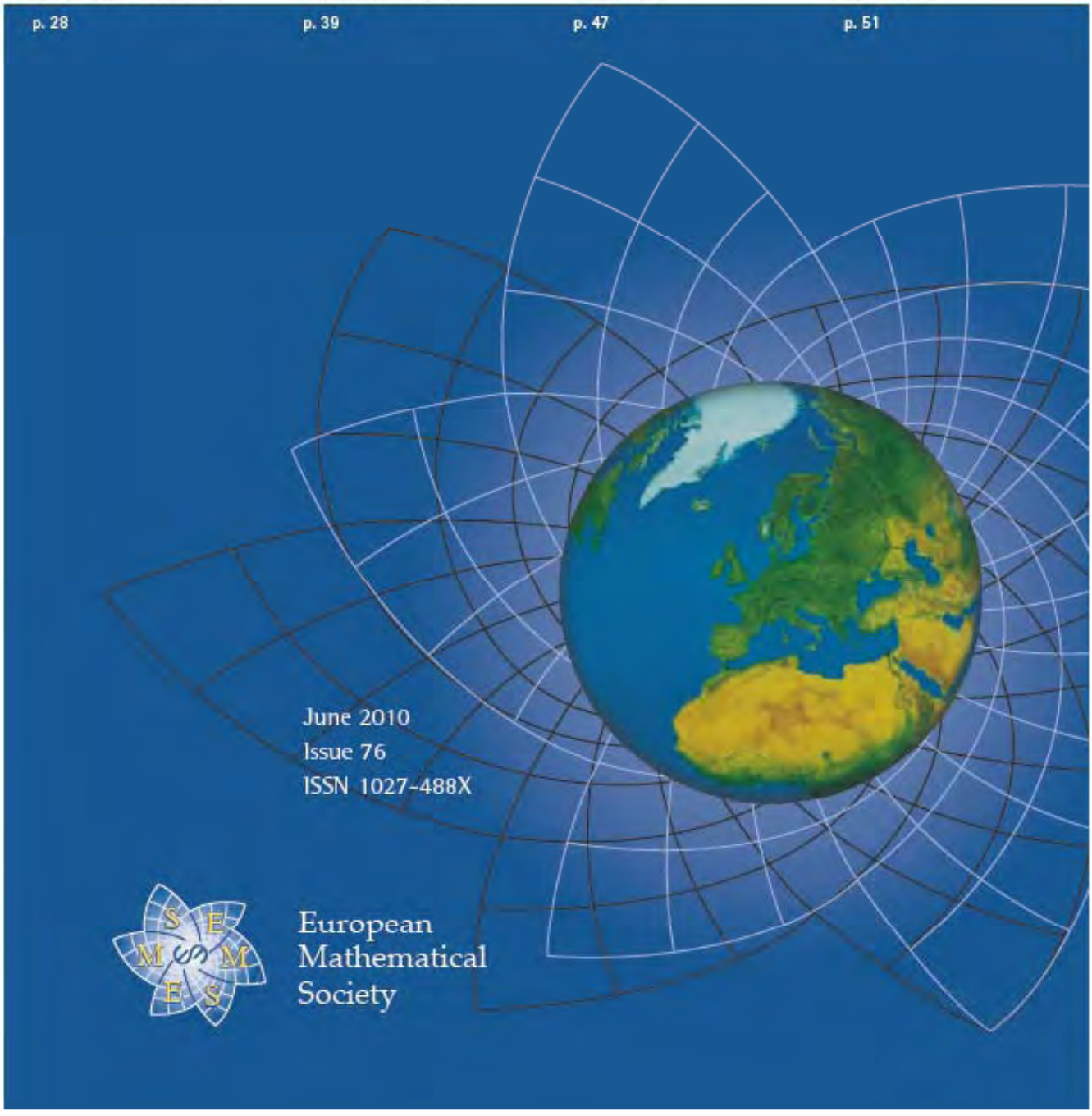

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It began with Pestalozzi and Schleiermacher Reflections on the polymath Hermann Graßmann (1809–1877)

Hans-Joachim Petsche

"I therefore decided to make the presentation, extension, and application of this analysis my life task."

Graßmann, in the preface to his *Extension Theory* of 1844

His life's destiny: Graßmann's Ausdehnungslehre, a story of disappointment and success

In 1853, the brilliant Irish mathematician and physicist W. R. Hamilton (1805–1865) stumbled upon Graßmann's *Extension Theory* of 1844 (hereafter, A1) while doing historical research for the introduction to his *Lectures on Quaternions* [19]. Graßmann's book impressed Hamilton greatly. In a letter to his friend De Morgen, dated 31 January 1853, he wrote:

William Rowan Hamilton (1805–1865).

"I have recently been reading *[…] more than a hundred pages of* Graßmann's Ausdehnungslehre*, with great admiration and interest. […] If I could hope to be put in rivalship with Des Cartes on the one hand, and with Graßmann on the other, my scientific ambition would be fulfilled!" [18, 441]*

Hamilton's enthusiasm contrasted greatly with the reactions that Graßmann's main mathematical work had provoked up to that point. Later, Moritz Cantor (1829–1920) would dryly remark:

> *"In 1844, O. Wigand published the book in Leipzig. Nobody reviewed it, nobody bought it, and almost all of the copies of that edition were destroyed." [3,569]*


Die

lineale Ausdehnungslehre

ein

neuer Zweig der Mathematik

Hermann Grassmann

Leipzig, 1844.

Conscious of the fact that he was providing the basis for a completely new approach, Graßmann had relied on *conceptual construction* in order to create an algebraic theory of *n-dimensional* manifolds in his *Extension Theory*. These manifolds were completely disconnected from geometrical *sense-perception.* By doing so, his theory provided its own *philosophical* foundation; it began by presenting an abstract *general theory of forms* of conjunctions, underlying the entire structure of mathematics, and continued to develop its theoretical object – linear and multilinear algebra of *n*-dimensional space – by closely connecting mathematical and conceptual constructions to *philosophical and heuristic* reflections.

By taking a look at Wilhelm Traugott Krug's *General Handbook of the Philosophical Sciences* of 1827, we can fathom to what point Graßmann's approach contradicted scientific zeitgeist at the time. For the term "mathematics", we find the following entry:

> *"Mathematics […] only [deals with] magnitudes which appear in time and space […] A philosopher should familiarize himself with mathematics and a mathematician with philosophy, as far as their talent, interests, time and surroundings will permit. But one should not confuse and throw into one pot what the progress of scientific knowledge has separated, and rightly so. […] mathematical philosophy and philosophical mathematics – in the commonly accepted sense of the terms, namely as a mixture of both – are scientific or, rather, unscientific monsters." [21]*

When Graßmann's contemporaries, the mathematicians Gauß (1777–1855), Apelt (1812–1859), Grunert (1797–1872) and Balzer (1818–1887), rejected A1, they were following this Kantian view [see 27, ix sqq.].

However, Graßmann's philosophical work had not compromised the autonomous integrity of the concept of mathematics. Rather, by disconnecting it from sense-perception, he had contributed to the establishment of *pure mathematics*.

In 1862, Graßmann presented a completely revised version of his *Extension Theory* [9] (hereafter, A2). Reluctantly, he had removed all philosophical considerations from the text. Now the book approached its object by developing a string of formulas, making it just as readable to us as any contemporary mathematical treatise. Nevertheless, at the time, the majority of the mathematical readership did not accept it.

Ernst Abbe (1840–1905).

It took a new generation of mathematicians, Hermann Hankel (born 1839), Ernst Abbe (born 1840), Josiah Willard Gibbs (born 1839) and Felix Klein (born 1849), before Graßmann's ideas found approval in different contexts.

As a young man, Ernst Abbe (1840–1905) had attended Riemann's

lectures. Hermann Hankel (1839–1873) had relied on Abbe's manuscripts from the lectures [see 1, xxiii]. Abbe, whose work in optics contributed to the enormous entrepreneurial success of the Carl Zeiss Company for optical instruments, had encountered A1 in 1861 and was enthusiastic about the book's "heuristic form" of presentation. In June 1862, informing his friend Harald Schütz (1840–1915) about his findings in Graßmann's texts, he wrote:[1]

> *"I must say that this is very enjoyable work. This man has certainly produced a brilliant piece of work, and I find it hard to believe that it has remained completely unknown. My satisfaction grows larger with every page I read. The more I manage to penetrate and understand this utterly unique and in fact quite un-mathematical way of looking at things, the more I realize how adequate and justified it really is. […] But this is not merely about shedding light on the philosophical background of mathematics. Rather, the author convincingly demonstrates that there really are things to be gained. He applies the principles of his 'new analysis' to a large number of actual problems – be they geometrical, mechanical or purely arithmetical – and thereby uncovers completely new methods everywhere, which produce new theorems in an incredibly efficient and transparent way. And even if we should already be familiar with these theorems, our procedures of generating them presuppose a long and complicated list of concepts and formulas." [1, 214–215]*

Unfortunately, Graßmann never learned of Abbe's positive reaction to his work. Despite the fact that a university professorship would have meant a certain loss of income, Graßmann applied more than once for such a position – in vain. Graßmann would always remain a teacher in Stettin. Disappointed by mathematics, he turned to studies of philology, which he had once considered mere recreation from his mathematical efforts. His works on the ancient Indian hymns of the *Rig-Veda* [10], [11] are important references even today, and Graßmann almost immediately received attention from philologers of Sanskrit.

Graßmann's letter of thanks upon becoming a member of the Göttingen Society for the Sciences.

It was late in his life when Graßmann finally received the acclaim he had longed for. On 2 December 1871, thanks to the initiative of Alfred Clebsch (1833–1872), he was made a member of the Göttingen Society for the Sciences.

A second edition of A1, which Graßmann was still able to prepare, was published in 1878, a year after his death. Beginning in 1876/77, Ernst Abbe started giving lectures on Graßmann's calculus [see 30, 200]. Rudolf Mehmke (1857–1944) lectured on *Extension Theory* in 1881/82 and the following years [see 29]. As a young man, in 1888, Alfred North Whitehead (1861–1947) also began his career as a university lecturer by addressing Graßmann's *Extension Theory*. In 1898, Whitehead published his *Universal Algebra*. In the introduction, he wrote:

Alfred North Whitehead (1861–1947).

"The greatness of my obligations in this volume to Grassmann will be understood by those who have mastered his two Ausdehnungslehres. The technical development of the subject is inspired chiefly by his work of 1862, but the underlying ideas follow the work of 1844." [39, x]

Inspired by Josiah Willard Gibbs (1839–1903), Felix Klein (1849–1925) began to demand an edition of Graßmann's works in mathematics and the natural sciences. Edited by Friedrich Engel (1861–1941), the five volumes of Graßmann's collected works appeared between 1894 and 1911 (volume 6 was reserved for Friedrich Engel's biography of Graßmann [6]).

At this point, the reception of Graßmann's *Extension Theory* was at a peak. Graßmann's book inspired an incredibly large number of scientists. Those that come to mind are: in mathematics – Klein, Mehmke, Peano, Veronese, Gibbs, Heaviside and Whitehead; in philosophy – Natorp, Cassirer, Kuntze, Husserl, Carus and many others; and in psychology – most notably Preyer and Wundt.

The third International Congress of Philosophy took place in 1908. On the evening preceding the 100th anniversary of Graßmann's birth, Friedrich Kuntze (1881–1929) held a lecture on the "significance of Hermann Graßmann's *Extension Theory* for transcendental philosophy". He concluded his talk with the following words:

> *"While Aristotle was the first to isolate pure forms of thinking,* Graßmann was the first to isolate the pure forms of sense-perception*. […] So if Graßmann's* achievements *are in the vicinity of Aristotle, why not also place his* name *next to Aristotle's? And herewith I respectfully and affectionately dedicate these words to the memory of a great man, who remained unrecognized during his days on earth and who has all the right to expect from history the kind of justice that life denied him." [22, 437]*

Roots of Graßmann's creativity

Whenever we encounter a brilliant scientist like Hermann Graßmann, we automatically pose questions that aim to understand the general aspects of these unique creative eruptions in the history of science. But, at first sight, Graßmann seems like a difficult case:

- Young Hamilton, whose theory of quaternions was closely linked to Graßmann's external algebra, was a complete prodigy. Hamilton could boast that at age 13 he had mastered 13 languages (among them Arabic

[1] Letter from Ernst Abbe to Harald Schütz, 21 June 1862.

and Sanskrit), while young Graßmann was a dreamer, easily distracted and quite sluggish. Apparently, Graßmann's father often assured his son that he would bear it stoically should the boy's career remain limited to garden work.

- Bernhard Riemann (1826–1866), who – like Graßmann – introduced the concept of n-dimensional manifolds to geometry, had teachers as prominent as Moritz Stern (1807–1894), Johann Benedict Listing (1808–1882), Carl Friedrich Gauß (1777–1855), Peter Gustav Dirichlet (1805–1859), Carl Gustav Jakob Jacobi (1804–1851) and Gotthold Eisenstein (1823–1852). Gauß "provoked" Riemann to choose a groundbreaking topic for the lecture he gave for his habilitation. Graßmann was a different case altogether. During his studies of theology in Berlin, he did not attend a single course in mathematics. His personal library contained only a small number of mathematical tracts. Graßmann mostly learned about mathematics from his father's works.
- Graßmann was never part of the university world. Apart from a one-year episode at the Berlin School of Commerce, Graßmann worked as a secondary school teacher in Stettin, separated and isolated from the centres of mathematical research. It was here that he wrote his masterpiece, the *Extension Theory* of 1844. Communicating sporadically with August Ferdinand Möbius (1790–1863) and collaborating intensely with his brother Robert, Graßmann continued to build his mathematical theories in the following years.

So, if at first glance Graßmann's achievements seem disconnected from the scientific ambitions of his time, a second glance will show us that this is not at all the case.

Graßmann's personality was the focal point (despite being on the sidelines of contemporary mathematical research) in which the most diverse influences and predispositions amalgamated, interacted in different ways and propelled him towards conclusions that were decades ahead of his time.

We can quite easily identify a set of conditions that must have transformed apparent disadvantages into mathematical creative energy. In an attempt to summarize these elusive factors in a few points, we might say:

1. In the first half of the 19th century, Stettin went through a phase of provincial and petit-bourgeois prosperity. This heightened the local academics' awareness of their personal creative powers, an insight that also drove Hermann Graßmann.
2. At the time, Prussia was reforming its systems of elementary schools and schools for the poor (Pestalozzi/Schleiermacher, see [24], [26]). This created pedagogical impulses aimed at revising the foundations of mathematics, which affected both Hermann Graßmann and his father.
3. Pestalozzi's collaborator Joseph Schmid (1785–1851) had created a theory of forms for elementary school purposes [33]. It was this project that provided the initial impulse for many of the ideas underlying and supporting *Extension Theory* (Graßmann algebra, the general theory of forms).
4. Schleiermacher's *Dialectic* [31] is largely responsible for the structure and mechanisms that turned a theoretical vision into a mathematical theory.

Graßmann reinvents himself

Young Graßmann was astonishingly open in the official curricula he wrote, describing his inner struggle against "phlegm", which he was aiming to "eradicate completely" in order to attain personal perfection. He attempted to change his way of life, overdid it and failed, finally developing a more moderate view of himself that turned weaknesses into strength: "Thus the Phlegmatic must not aspire to force the inefficiency of his thoughts into a bold flight; for the wing upon which he strives to soar to the sun is not proper to himself; and he will, like Icarus, soon plunge back to earth. Rather, he must seek to give his train of thought clarity, and in clarity depth."[2] [26, 146] For a long time, Graßmann was undecided about whether to become a theologian or a teacher. The ideas forming the point of departure of his *Extension Theory* only came to him in his *Theory of Tides*, which he wrote for his second round of exams on the way to becoming a teacher. He would dedicate the rest of his life to developing these ideas. When he described himself as a phlegmatic person in a letter to his fiancée in November 1848, Graßmann linked phlegm to persistency:

Hermann Graßmann (pencil drawing by a pupil).

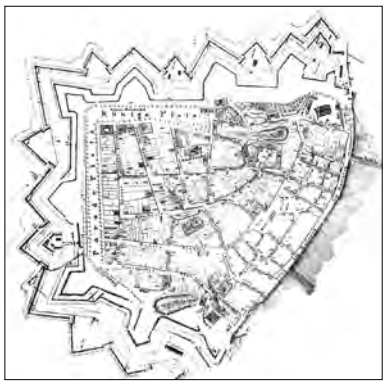
"He finishes what he begins. If he doesn't succeed on the first try, he begins anew and perseveres until he has finished the task. He focuses on what is immediately before him. He subordinates all of his deeds and goals to the overall view of the whole, to the single idea which he has identified as his life's task and inner motivation. Therefore his entire life and work form a whole, and everything around him becomes a part of it. His will is strong, his endeavours well-planned. He pursues his goals patiently and remains absolutely true to his life's task." [quoted from 6, 150]

Even after Graßmann had to face the failure of his dream that Extension Theory might immediately influence the further course of mathematics, at more than 50 years of age he nevertheless mustered the energy that had once turned an insecure young man into a scholar. For years, Graßmann sat down every day to do the painstakingly precise work of composing his monumental dictionary of the ancient Indian *Rig-Veda* [10] – apart from teaching in school for 18 to 20 hours a week.

Graßmann and Stettin

Following the War of Liberation of 1813/14, the town of Stettin experienced a phase of petit-bourgeois and

[2] Life history of Hermann Graßmann (23 March 1833) on the occasion of his theological examination. [26, 136–149]



The fortified town of Stettin around 1847.

provincial prosperity, which came to an end in the 1850s, when the railroad connection to Berlin was completed and the bourgeois revolution of 1848 had taken its toll.

During these four decades, romanticism, religiosity and German nationalism thrived within the city walls of Stettin. The petit-bourgeois quest for knowledge and education was underway and the Freemasons experienced an unprecedented upswing in membership. The Stettin secondary school, or "Gymnasium", represented the city's scientific and cultural centre [see 38]. It was home to a faculty of professors – some of them brilliant scientists, others narrow-minded locals – who did not accept any scientific authority as long as it did not live up to their own standards. The only thing that united this highly diverse group of academics was the mindset of romanticism. This was a micro-climate that relied on the creative powers of individuals and that provided the soil for Hermann Graßmann's gradually growing self-confidence. It was here that he started to believe that he could completely revise the structure of mathematics, a project that was to begin with a completely new branch of mathematics: extension theory. Shortly after Hermann's return to Stettin from his one-year stint at the Berlin School of Commerce, he told his brother Robert:

Lodge Garden in Stettin.

"I have now already been a month and a half in Stettin, and I cannot tell you how well it goes with me here, especially in comparison with Berlin. […] The stimuli here are not so stormy but all the more heartfelt; the circle of activity in the calling not so great, but for all that more beneficial for those to whom it is directed, as for me myself; and it is true that for myself

Hermann Graßmann's life

1809–15 April:	Graßmann is born in Stettin.
1827	Hermann Graßmann begins his studies of theology at the University of Berlin.
1834–36	Graßmann obtains a position at the Berlin School of Commerce. After 14 months he gives up and returns to Stettin. He becomes a teacher at the "Ottoschule". He will never leave Stettin again.
1838/39	Graßmann signs up for the second examination in theology and asks the examination commission in Berlin for a second round of examinations in physics and mathematics.
1840	Graßmann submits the examination thesis on low and high tides to the commission in Berlin. *Extension Theory* is born.
1840	Hermann and Robert Graßmann study Schleiermacher's *Dialectic*.
1843	In the autumn Graßmann finishes work on the first volume of *Extension Theory* (A1). Hamilton discovers quaternions.
1846	Graßmann is awarded a prize honouring Leibniz for his *Geometrical Analysis*. Ideas from Extension Theory receive public recognition for the first time.
1847	Graßmann attempts to get a professorship in mathematics at a university. Kummer's assessment destroys Graßmann's hopes.
1852	Graßmann's father dies. In July, Graßmann becomes his father's successor at the Stettin "Gymnasium".
1860	Graßmann publishes his *Teaching Manual of Arithmetic*. That same year, he publishes his first article on philology.
1861	The completely revised and restructured version of *Extension Theory* is published.
1862	For the second time, Hermann Graßmann officially applies for a professorship in mathematics. The project fails. He feels disappointed with mathematics and focuses exclusively on Sanskrit and *Rig-Veda*.
1866	Graßmann begins an exchange of letters with Hankel.
1869	Klein discovers Graßmann by reading Hankel.
1869	Graßmann's oldest son sets out to study mathematics in Göttingen. He brings Clebsch and Stern a copy of Extension Theory.
1871	On Clebsch's initiative Graßmann becomes a member of the Göttingen Science Society.
1876	Graßmann is made a member of the American Oriental Society. On Roth's initiative, the University of Tübingen accords Graßmann an honorary doctorate for his philological work.
1877	Hermann Günther Graßmann dies on 26 September.
1894–1911	Publication of Graßmann's collected works.

the resources here do not flow as extensively, but I am all the more in a position to use them." [26,161–163]

Graßmann's personality arose from a peculiar combination of local convictions and global zeitgeist, linked to provincial resistance to the rapid and self-centred development of modern science. Walking a thin line between provincialism and German nationalism, on the one hand, and scientific creativity and brilliance, on the other, Graßmann mirrored the academic milieu surrounding him (though his brother Robert Grassmann (1815–1901) was even more affected by his surroundings).

Felix Klein was already aware of the fact that a certain degree of distance from institutionalized scientific research could produce considerable potential for theoretical force and depth:

"We academics grow in strong competition with each other, like a tree in the middle of the forest which must stay slender and rise above the others simply to exist and conquer its portion of light and air. But he who stands alone, like Grassmann, can grow on all sides, can harmoniously develop and finish his nature and work. Of course such versatility as Grassmann embodied must inevitably be accompanied by a certain amount of dilettantism…" [20, 161]

Graßmann's father

The introduction to A1 is the only instance in which Hermann Graßmann pointed to the inspiration he had received from his father Justus Graßmann (1779–1852).

Justus Graßmann (1779–1852).

"While I was pursuing the concept of the product *in geometry as it had been established by my father, I concluded that not only rectangles but also parallelograms in general may be regarded as products of an adjacent pair of their sides […]" [7, 9; emphasis H.–J. P.]*

Hermann Graßmann was referring to ideas from textbooks for elementary schools and lower grades in secondary schools. Most notably, the *Geometry* ("Raumlehre") of 1817 tells us:

"The concept of the present book was born with a number of schools for the poor, which I helped establish […] with no pay whatsoever and for the sake of doing good deeds." [13, iii]

Following Friedrich Schleiermacher (1768–1834) and Wilhelm von Humboldt (1767–1835), who considered the rebirth of the school system a prerequisite for the national rebirth of Germany after the Napoleonic occupation, Justus Graßmann collaborated with his brother Friedrich Heinrich Gotthilf Grassmann (1784–1866) and schools councillor Georg Wilhelm Bartholdy (1765–1815) – a close friend of Schleiermacher – on textbooks that were inspired by the pedagogical methods of Johann Heinrich Pestalozzi (1746–1827). These textbooks were meant to serve as free material for teachers "lacking a scientific education in the strict sense of the word". As it turned out, the three collaborators were highly successful in their project.

This is how Wilhelm Dilthey (1833–1911) put it:

Detail of the illustration "Pestalozzi in Stanz" (1845). The publisher and critic Diesterweg took note of Pestalozzi's followers in the right column, among them Friedrich Heinrich Gotthilf Grassmann.

"The collaboration between the excellent school councilor Bartholdy, the teacher Graßmann and his brother had created a lively pedagogical movement in Stettin. Without making any noise about it, they transformed a school for the poor into a model for other institutions…" [5, 479]

Justus Graßmann's discussions with Bartholdy did not remain confined to Pestalozzi's pedagogical methods (Schleiermacher remarked that he was totally in accord with Bartholdy's reception of Pestalozzi's ideas[3]). Bartholdy and Justus Graßmann probably also discussed Schleiermacher's first publications on the theory of science[4] while the latter was working on his schoolbooks on geometry, which relied on the theory of forms developed by a follower of Pestalozzi, Joseph Schmid. In these schoolbooks, he developed the outlines of a "geometrical theory of combinations". In 1829 [12], he applied it to crystallography and presented his theory to the scientifically educated public. By thinking about an adequate way to make geometry accessible to children, Justus Graßmann started to think about ways of presenting geometry that deviated from the methods of Euclid. This, in turn, led him to reflect upon the foundations of mathematics.

"Methodological work on the pedagogical subjects in question" had to entail "clarity as to which elements make up these pedagogical subjects," Justus Graßmann wrote in a textbook of 1817. "Only by establishing the primordial

[3] Schleiermacher about Bartholdy: "In Berlin, he told me about his plans for a seminar, which made me very happy, and from which I conclude that I share his view of Pestalozzi's idea and its essential importance." Letter from Schleiermacher to Joachim Christian Gaß (1766–1831), May 1805. [32, 23]

[4] Copies of the first lectures of Schleiermacher in Halle began to circulate amongst interested readers. Bartholdy also received a copy. He studied Schleiermacher's text carefully with Gaß, copied it down for himself and spoke very positively about Schleiermacher's ideas. According to a letter from Gaß to Schleiermacher from July 1805, the lectures on philosophical ethics contained a detailed explanation of Schleiermacher's transcendental postulates. Here he developed his own position as an alternative to Schelling's. (Letter from J. Chr. Gaß to Schleiermacher, 13 July 1805 [32, 25sq.], and also [27, 244sqq].)

Hermann Graßmann as a teacher

As Max Ludewig – who graduated from Stettin "Gymnasium" in 1865 – remembered him, Hermann Graßmann's behaviour as a teacher was not overly scholarly. But "concerning the demands of everyday life, he seemed like a benevolent child. His eyes were hidden behind spectacles, but generous and friendly. He left it to his favourite pupil to maintain order in class. He never reprimanded us. Usually, he stood by the first row of desks. We smeared chalk on the buttons of his vest, without him noticing. If and when he noticed something worthy of his disapproval, he merely murmured 'now, now!' But we still learned a lot from him because his thinking was full of scientific clarity."

(M. Ludewig, *Erinnerungen eines alten Stettiners*. Stettin 1918)


Raumlehre
für Volksschulen.
Von
J. G. Graßmann,
Erster Theil.
Ebene räumliche Verbindungslehre.
Mit 5 Kupfertafeln.
Berlin, 1847.

prerequisites of the subject in question, from a scientific or a classroom perspective, are we able to establish the true and lasting importance of what we are dealing with…" [13, viii]

While he was writing his textbooks of geometry, which received favourable reviews from Friedrich Adolf Wilhelm Diesterweg (1790–1866) [4: vol.1, 195–197, 264, 525; vol. 3, 224], Justus Graßmann was already aware of the fact that the geometric theory of combinations was a *new mathematical discipline*. It is here that we find the first reflections concerning the essence of mathematical synthesis, the dialectic structure of mathematics and the organic integrity of theory-building that supposedly made theory "analogous to a work of art". In his treatise *On physical crystallonomy and the geometric theory of combinations* [12], Justus Graßmann developed his mathematical theory. This is how Erhard Scholz described it:

> *"In modern terminology, Justus Grassmann introduced into his 'calculus of complexions' a three-dimensional free Z-module, whose elements […] represented directed lines of Euclidean space in an algebraic symbolism. He thus introduced with great clarity a three-dimensional vectorial calculus with integer coefficients." [34, 41]*

The ideas of father and son were closely connected and we might say that Hermann Graßmann completed and perfected what his father had begun. Other publications offer a detailed account of this relationship.

Hermann Graßmann had referred to his father's *concept of the product* in his *Extension Theory*. But he had understated the case when he wrote that the influence remained confined to the surfaces of rectangles or parallelograms as products of the adjacent sides. In a footnote to his textbook on trigonometry (1835), to which Hermann also referred [7, 9], Justus Graßmann wrote:

> *"If one takes the concept of product in its purest and most general meaning, then – in mathematics – the concept designates the result of a synthesis. This synthesis uses the entities produced by a preceding synthesis, replaces these entities, but maintains the rules governing the initial entities. […] In arithmetic, this entity is an element […]. In geometry, this element is a point." [14, 10 footnote]*

This way of putting it makes it easy to discern Graßmann's abstract approach to mathematical conjunctions, an approach that also expressed itself in the general theory of forms, preceding the main corpus of A1. As a genetic and constructive program, this led to the first *abstract* algebraic theory. In L. G. Biryukova and B. V. Biryukov's words: "In such of Hermann Grassmann's works as the Ausdehnungslehre is to be found the definition of an abstract group (ten years before Cayley's work on groups), and the concept of ring is developed, yielding both left and right rings." [2, 137]

Schleiermacher

The hints concerning the influence of the theologian and philosopher Friedrich Schleiermacher are just as elusive as those concerning Graßmann's father. In the curriculum vitae that he submitted for an examination in theology at the University of Berlin in March 1833, Graßmann wrote:


Friedrich Schleiermacher (1768–1834).

"Yet only in the last year did Schleiermacher attract me completely; and although by that time I was more concerned with philology, still only then did I realize what one can learn from Schleiermacher for every science, since he did not so much provide positive answers, as he made one skilled in attacking every investigation from the correct side and continuing independently, and thus to stand in a position to find the positive answer oneself." [26, 145–146]

Robert Grassmann's writings support this view. In an obituary for his brother, published only recently, he wrote that Schleiermacher's "ingenious dialectical method exerted the greatest influence on the young Grassmann." [26, 203sq.]

One of Hermann's sons Justus Graßmann (1851–1909) also wrote a biographical summary of his father's life. This recently discovered document also reveals information on Schleiermacher's influence. Justus wrote that, as a student, his father "seems to have been drawn especially close to

Schleiermacher, on whose lectures and works many commentaries were found among the papers he left behind." [25] Unfortunately, this collection of papers on questions concerning the humanities and philosophy remains lost, like the papers on mathematics and the natural sciences. The two brothers read and discussed Schleiermacher's *Dialectic* (1840), his *Aesthetic* (1845) and, finally, his *Theory of the State* (1847).

In the *Edifice of Knowledge*, Robert Grassmann repeatedly emphasised how important he considered Schleiermacher to be, saying that Schleiermacher was the "*most important […] critic we have seen in recent times.*" [15, 82] "*Schleiermacher's great merit*," he explained, "*is that he was the first to truly grasp and introduce into science a theory of scientific discovery or speculation as the highest branch of the science of logic. He has this merit even though his idea remained stuck in a theoretical stage and even though Schleiermacher did not yet know how to use it to reorganize the sciences.*" [15, 82–83]

And Robert went on to say:

> *"According to his 'Dialectic' Berlin 1839, only two academic fields can show us the idea of knowledge. Both of these fields deal with the idea of knowledge, that is to say, the mutual relationships between thinking and existence. Dialectics, which deals with the oppositions within unity, does so in the conceptual frame of the general, whereas mathematics, which only deals with equal and unequal magnitudes, does so in the conceptual frame of the particular. According to him, all true thinking is scientific thinking depending on to what extent dialectics and mathematics are a part of it (§§344–346). Mathematics is closer to the empirical, dialectics closer to the speculative form. The empirical process always precedes the speculative process, contextualizing it. Schleiermacher is completely on the mark in these theorems; but, as he remarked himself, he lacked knowledge of mathematics." [15, 83]*

In September 2009, historian of mathematics Ivor Grattan-Guinness broke new ground by pointing out the fact that Schleiermacher's Dialectic had provided the foundational structure for Robert Grassmann's life's work – the *Edifice of Knowledge* [see 17]. Albert C. Lewis [23] and H.-J. Petsche [27, 244–248; 28] have shown that just this approach serves to unfold the new mathematical theories of Graßmann's *Extension Theory* of 1844 [see 16].[5]

Graßmann's *Extension Theory* is one of the rare and precious cases in which a philosophical theory puts its heuristic potential into action and spawns new theoretical insights. On the other hand, it was unfortunate for Graßmann that mathematicians at the time could not accept A1 due to their reservations concerning the philosophical discussions that were underway.

Graßmann, the polymath

Even though the two versions of *Extension Theory* are clearly the creative core of Graßmann's work, his total intellectual output was far more diverse. His T*extbook of Arithmetic* [8] was an important impulse for the development of axiomatic mathematics, inspiring Peano, Frege and many others. Graßmann was also noted for his discoveries in the theory of electricity (where his work had some points of contact with Clausius), and the theories of colour and of vowels. Graßmann made amendments to Helmholtz's theory and extended it. He published works on the didactics of experiments in the teaching of chemistry and on crystallography. He was also a pioneer of comparative philology and Vedaic research. A sixth, revised edition of Graßmann's dictionary of the *Rig-Veda*, a collection of pre-Buddhist divine hymns from India (12^{th}–6^{th} centuries BC) was published in 1996 [10]. Graßmann's translation of *Rig-Veda* is the authoritative translation to this day.

> *"Indeed I well know," Graßmann wrote in the last lines of his introduction to the second version of the* Extension Theory *of 1862, "that the form I have given the science is, and must be, imperfect. But I also know and must declare, even at the risk of sounding presumptuous, – I know, that […] a time will come when it will be drawn forth from the dust of oblivion and the ideas laid down here will bear fruit." [9, xvii]*

Celebrating the occasion of Graßmann's birth 200 years ago, an international conference took place in Potsdam and Sczcecin in September 2009. For four days, over 75 scientists from four continents – among them a Chinese crystallographer, an Austrian robotics-engineer, an Australian software-specialist, a Japanese philologer, a Finnish philosopher, a Russian mathematician, a German hardware-architect, a Polish historian and many others – engaged in a transdisciplinary dialogue, following the traces of Graßmann's achievements and renewing their importance for scientific research today.

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[5] That is to say that Gert Schubring's view, which he stated in 1996 [36] and renewed in 2009 [37], that the philosopher Jakob Friedrich Fries (1773–1843) influenced Hermann Grassmann through his brother Robert, cannot be confirmed. On the contrary, there can be no doubt that Hermann communicated the thinking of Friedrich Schleiermacher to Robert Grassmann and that the methodological vision of Robert's thousands of pages of the "Edifice of Knowledge" bore its imprint, as Ivor Grattan-Guinness pointed out [17]. Fries is not mentioned in any of Hermann or Robert Grassmann's writings. Robert Graßmann also composed a 120-page History of Philosophy [15], in which he did not mention Fries. Schleiermacher, though, receives much acclaim and is hailed as the "most important […] critic we have seen in recent times". [15, 82]

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